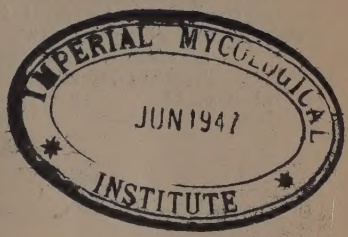
COMPTES RENDUS

(DOKLADY)

DE L'ACADÉMIE DES SCIENCES DE L'URSS

NOUVELLE SÉRIE

1945



VOLUME XLVII

№ 2

ÉDITION DE L'ACADÉMIE DES SCIENCES DE L'URSS
MOSCOU

AVIS AUX AUTEURS

Les «Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS» paraissent tous les dix jours. Neuf numéros composent un volume pourvu d'une table des matières. Il y a quatre volumes par année.

Les «Comptes Rendus de l'Académie des Sciences de l'URSS» en de courtes communications précisent l'essentiel des travaux en préparation. Ces articles, ne dépassant pas en règle générale quatre pages, sont donnés en anglais, en français ou en allemand. Ils embrassent les sciences mathématiques, physiques, naturelles et appliquées, mettant au jour les résultats de recherches scientifiques en cours ou qui viennent d'être achevées, et représentant ainsi les plus récentes données de l'investigation scientifique.

L'auteur a droit à 50 exemplaires de son article.

Adressez les manuscrits à la Rédaction des «Comptes Rendus», Moscou, Wolkhonka, 14.

20 Avril

COMPTES RENDUS

(DOKLADY)

DE L'ACADÉMIE DES SCIENCES DE L'URSS

COMITÉ DE RÉDACTION

A. Arbusov, de l'Académie, **V. Chlopin**, de l'Académie, **A. Deborin**, de l'Académie, **A. Fersman**, de l'Académie, **A. Frumkin**, de l'Académie, **A. Kolmogoroff**, de l'Académie (vice rédacteur), **L. Orbeli**, de l'Académie (rédacteur en chef), **A. Richter**, de l'Académie, **S. Soboleff**, de l'Académie, **S. Wawilow**, de l'Académie (vice rédacteur)

NOUVELLE SÉRIE

13-me année

Paraissant tous les dix jours

1945

VOLUME XLVII, № 2

TABLE DES MATIÈRES

Pages

MATHÉMATIQUES

M. M. Grinblum. Biorthogonal Systems in Banach Space 75

V. M. Dubrovsky. On Purely Discontinuous Random Processes with Residual Effect 79

P. V. Nikolaev. On the Anamorphosis of Symmetric Equations 82

THÉORIE DE L'ÉLASTICITÉ

J. N. Rabotnov. Fundamental Equations of the Theory of Shells 87

HYDRODYNAMIQUE

L. I. Sedov. On Unsteady Motions of a Compressible Fluid 91

PHYSIQUE MATHÉMATIQUE

L. I. Gutenmacher. Artificial Many-Dimensional Models for Approximate Solutions of Integral Equations 94

PHYSIQUE

M. I. Batuev. A Study of the Intramolecular Hydrogen Bond of Guaiacol by the Method of the Combination Scattering of Light. III 97

J. P. Terlezky. On Currents of Fast Charged Particles Induced by Revolving Magnetized Cosmic Bodies 101

CHIMIE

A. I. Yurzhenko and **S. M. Mintz.** Solubility of Polymerizing Hydrocarbons (Monomers) in Aqueous Solutions of Emulsifying Agents 103

CHIMIE PHYSIQUE

S. N. Zhurkov and **R. I. Lerman.** Influence of the Volume Sorption of Vapours on the Solidification Temperature of Polymers 106

S. G. Mokrushin. Formation of Monomicellar Films of Colloidal Hydroxides of Metals on the Surface of Hydrosols 110

AGROCHIMIE

M. M. Okunzov. Pb Content in Different Parts of the Caryopsis of Spring-Wheat and in the Plant as a Whole 113

CHIMIE BIOLOGIQUE

N. A. Maximov, Corresponding Member of the Academy, **G. V. Rakitin** and **R. Kh. Turezkaja.** An Attempt to Study Methods for Cultivating Gladioli as a New Raw Stuff for the Production of Vitamin C 116

GÉOLOGIE

B. L. Lichkov. Sedimentation, Denudation and Changes in the Organic World 120

MINÉRALOGIE

M. F. Beliakov. Lazulite from a Quartz Vein in the Region of Mount Suraiz in the Near-Polar Urals 124

G. I. Bushinsky. On the Classification of Phosphorites 127

I. Sedletzky and **A. Malovichko.** Rôle of Colloidal-Disperse Minerals in the Processes of Ground Freezing 130

CYTOLOGIE

P. V. Makarov. Demonstration of Paranecrotic Cell Alterations on Permanent Microscopic Preparations. Experiments on *Allium Cepa* and *A. sativum* 134

HISTOLOGIE

A. A. Braun and **G. N. Orlova.** Experiments on Heterotopic Transplantation of the Skin 138

PHYSIOLOGIE VÉGÉTALE

I. N. Holubinsky. Variation in the Content of Bitter Principle in the Fruits of Hop as Induced by pre-Treatment of Cuttings with Salt Solutions . . 140

A. A. Nichiporovich. Laticiferous System as One Regulating the Course and Direction of Biochemical Processes in Plants 142

M. Ch. Čajlachjan. Flowering in Different Plant Species as a Response to Nitrogenous Food 146

PHYSIOLOGIE

A. M. Damir. Evaluation of the Amount of Blood Displaced in the Experiment of Valsalva 150

Traductions rédigées par *D. Rakhmanov* et *T. Rogalina*

Édition de l'Académie des Sciences de l'URSS
Moscou 1945

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

MATHEMATICS

BIORTHOGONAL SYSTEMS IN BANACH SPACE

By M. M. GRINBLUM

(Communicated by A. N. Kolmogoroff, Member of the Academy, 22. VI. 1944)

Let E be a separable Banach space (1). A sequence of elements $\{x_i\}$ in E is said to be complete in E if the closed linear hull over $\{x_i\}$ coincides with E. A sequence $\{x_i\}$ is bounded if $\sup_i \|x_i\| < \infty$. A sequence $\{x_i\}$ is minimal (2) if no x_i belongs to the closed linear hull over the other elements of the sequence.

It is evident that if $\{x_i\}$ is minimal, there exists then a sequence of linear functionals $\{F_i\}$ on E which forms together with $\{x_i\}$ a biorthogonal system. If, moreover, $\{x_i\}$ is a complete sequence, then $\{F_i\}$ is determined uniquely.

A sequence $\{x_i\}$ is said to be strictly minimal if there exists a constant $\delta > 0$ such that the distance from any x_i to the closed linear hull over the other $x_j\,(j \neq i)$ is not less than δ.

We say $\{x\}, \{F_i\}$ is a regular biorthogonal system if $\{x_i\}$ is a complete sequence and the following conditions are fulfilled: $\sup_i \|x_i\| < \infty$, $\sup_i |F_i| < \infty$.

A biorthogonal system $\{x_i\}$, $\{F_i\}$ is regular if and only if $\{x_i\}$ is a complete bounded, strictly minimal sequence. $\delta = (\sup_i |F_i|)^{-1}$ is thereby the maximal admissible value of δ.

A sequence $\{x_i\}$ is called basis of E if every element $x \in E$ can be uniquely represented in the form: $x = \sum_1^\infty \alpha_i x_i$. It is evident that a basis is a complete strictly minimal sequence.

The following notations will be used throughout the paper: I_n is the linear hull over the elements $x_1, x_2, \ldots, x_n$ of the sequence $\{x_i\}$; I^n is the closed linear hull over $x_{n+1}, x_{n+2} \ldots$; S_n is the surface of the unit sphere in I_n, *i. e.* the set of elements x satisfying the conditions: $x \in I_n$ and $\|x\| = 1$; S^n is the surface of the unit sphere in I^n.

§ 1. Let

$$\{x_i\}; \{F_i\} \tag{I}$$

be a biorthogonal system with $\{x\}$ complete and bounded.

Theorem 1. 1° *The system* (I) *is total if and only if*

$$\lim_{n\to\infty} F_n(x) = 0 \text{ for every } x \in E. \tag{1}$$

2° *The sequence* $\{F_i\}$ *in* (I) *is total if and only if the intersection of all manifolds* I^n *contains only the null element.*

The following result from our paper (²) will be useful:

Theorem 2. *In order that a complete sequence* $\{x_i\}$ *be a basis in* E, *it is necessary and sufficient that there exist a constant* $\alpha > 0$ *such that*

$$\rho(S_n, S^n) > \alpha \text{ for all } n$$

($\rho(S_n, S^n)$ *denotes the distance from* S_n *to* S^n).

§ 2. Let

$$\{x_i\}, \{\mathrm{F}_i\}$$

be a regular biorthogonal system and

$$\sup_i \| x_i \| = Q; \sup_i | \mathrm{F}_i | = M; \delta = \frac{1}{M} \tag{2}$$

Let further ε_n be the numbers defined by the formulae

$$\varepsilon_n = \frac{\delta - (\gamma_1 + \gamma_2 + \ldots + \gamma_n)}{2Q} \gamma_n, \tag{3}$$

where γ_n are any numbers satisfying the conditions

$$\gamma_n \geqslant 0, \quad \sum_1^\infty \delta_n < \gamma \tag{4}$$

All the ε_n are evidently non-negative.

Theorem 3. *Whatever be the elements* $\eta_1, \eta_2, \ldots$ *satisfying the conditions* $\| \eta_n \| < \varepsilon_n$ $(n = 1, 2, \ldots)$, *the sequence* $\{y_i = x_i + \eta_i\}$ *possesses an adjoint sequence* $\{F_i\}$ *and the biorthogonal system*

$$\{y_i\}, \{F\} \tag{II}$$

is regular. Thereby $|F| < 1/(\delta - \gamma)$ *where* $\gamma = \sum_1^\infty \gamma_i$.

Theorem 3′. *A sequence* $\{y = x_i + \eta_i\}$ *where* η_i *are arbitrary elements of* E, *which satisfy the condition*

$$\sum_1^\infty \| \eta_i \| < \frac{\delta^2}{8Q}$$

possesses an adjoint sequence $\{F_i\}$, *and the biorthogonal system* $\{y_i\}, \{F_i\}$ *is regular.*

Theorem 3′ follows immediately from Theorem 3. To verify this, it suffices to note that, for any sequence of positive numbers d_i satisfying the condition $\sum_1^\infty d_i < \delta^2/8Q$, a sequence of positive numbers $\gamma_1, \gamma_2, \ldots, \gamma_n, \ldots$ can be chosen such that $\sum_1^\infty \gamma_n < \delta$ and $d_n < \varepsilon_n$ $(n = 1, 2, \ldots)$, where ε_n are determined by the formulae (3).

Theorem 4. *If the sequence* $\{\mathrm{F}_n\}$ *in the regular system* (I) *is total, then the sequence* $\{F_i\}$ *in* (II) *is total as well.*

To every elements $x \in E$ there corresponds a sequence $\{\mathrm{F}_i(x)\}$ of the Fourier coefficients with respect to the system (I). Let $\mathfrak{A}_\mathrm{I}$ denote the set of all such sequences, while $\mathfrak{A}_\mathrm{II}$ denotes the corresponding set with respect to the system (II), *i. e.* the totality of sequences $\{F_i(x)\}$, $x \in E$.

Theorem 5. $\mathfrak{A}_\mathrm{I} \equiv \mathfrak{A}_\mathrm{II}$. *To put it differently, the system of equations*

$$\mathrm{F}_i(x) = F_i(y) \; (i = 1, 2, \ldots) \tag{5}$$

has a solution in x *for erery* $y \in E$, *and likewise, a solution in* y *for every* $x \in E$

If $\{F_i\}$ *is total, then the system of equation* (5) *determines a linear one-to-one mapping of* E *onto itself.*

For the sequences $\{F_i\}$ and $\{F_i\}$ the following theorems take place:

Theorem 6. *For any number* σ *there is a number* $\varepsilon > 0$ *such that, whenever* $\sum_1^\infty \|\eta_i\| < \varepsilon$, $|F_i - F_i| < \delta$ *uniformly in* i $(i = 1, 2, \ldots)$. *More accurately: if* $\|\eta_i\| < \varepsilon_i$ $(i = 1, 2, \ldots)$ *where* ε_i *are determined by* (3), *then for all* i

$$|F_i - F_i < \frac{\gamma}{\delta(\delta - \gamma)}$$

where $\gamma = \sum_1^\infty \gamma_i$ ($\varepsilon \to 0$ *implies* $\gamma \to 0$).

Theorem 7. $\lim_{n \to \infty} F_n - F_n| = 0$.

Remark. The series $\sum_1^\infty |F_n - F_n|$ is, in general, divergent. In case this series converges for any sequence satisfying the conditions $\|\eta_i\| < \varepsilon_i$ $(i = 1, 2, \ldots)$, the series $\sum_1^\infty |F_i(x)|$ converges for every $x \in E$.

§ 3. Denote by $\mathfrak{M}$ the set of all complete bounded strictly minimal sequences in E. The elements of $\mathfrak{M}$ will henceforth be denoted by $\xi, \zeta, \varkappa$. Thus, $\xi \equiv \{x_i^\xi\}$. In virtue of the completeness of $\{x_i^\xi\}$, to every element ξ there corresponds uniquely a sequence $\{F_i^\xi\}$ adjoint to $\{x_i^\xi\}$. Moreover, the number $\delta(\xi) = 1 \left(\sup_i |F_i^\xi|\right)$ and $Q(\xi) = \sup_i \|x_i^\xi\|$ correspond uniquely to every element $\xi \in \mathfrak{M}$. Introducing the notations S_n^ξ and $S^{n\xi}$ for the former S_n and S^n, we define the number $\alpha(\xi)$: $\alpha(\xi) = \inf_n \rho(S_n^\xi, S^{n\xi})$. If $\alpha(\xi) < 0$, then $\{x_i^\xi\}$ is a basis in E (see Theorem 2).

Keeping in mind Theorem 3, we may topologize $\mathfrak{M}$ in a natural way. We say that the element $\xi \equiv \{x_i^\xi\}$ possesses an ε-neighbourhood, if every sequence $\{y_i = x_i + \eta_i\}$ with η satisfying the condition $\sum_1^\infty \|\eta_i\| < \varepsilon$ is complete bounded and strictly minimal (*i. e.* is an element of $\mathfrak{M}$). According to the remark to Theorem 3′, *every element* ξ *of* $\mathfrak{M}$ *has an* ε*-neighbourhood, as soon as* $\varepsilon < [\delta(\xi)]^2 / 8Q(\xi)$.

Let the ε-neighbourhood of the element ξ be denoted by U_ξ^ε. We also shall consider U_ξ^ε as a neighbourhood of every element of $\mathfrak{M}$ contained in U_ξ^ε.

It is not difficult to establish the following facts:

A. The system of sets U_ξ^ε *possesses the properties that characterize a system of neighbourhoods in a topological space.*

$\mathfrak{M}$ will henceforth denote the topological space.

B. *The function* $\delta(\xi)$ *is continuous on* $\mathfrak{M}$.

Theorem 8. *The function* $\alpha(\xi)$ *is continuous on* $\mathfrak{M}$.

The proof of this theorem is based on Theorems 2 and 3. M. Krein, M. Rutman and D. Milman proved the following theorem (4):

If $\{x_i\}$ is a basis in E, then there is a sequence of positive numbers $\delta_1, \delta_2 \ldots$ such that the sequence $\{y_i = x_i + \eta_i\}$ is also a basis, whenever η_i satisfy the conditions $\|\eta_i\| < \delta_i$ $(i = 1, 2, \ldots)$.

Theorem 8 implies immediately a slightly improved result:

Theorem 9. *If* $\{x_i\}$ *is a basis in* E, *there exists* $\varepsilon > 0$ *such that, for any elements* $\eta_1, \eta_2, \ldots$ *satisfying the condition* $\sum_1^\infty \|\eta_i\| < \varepsilon_i$ *the sequence* $\{y_i = x_i + \eta_i\}$ *is a basis, too.*

We say that an element $\xi \in \mathfrak{M}$ can be connected with another element $\zeta \in \mathfrak{M}$ by a finite chain if there exists a finite sequence of elements $\xi_1 = \xi; \xi_2, \ldots \xi_{n-1}; \xi_n = \zeta$ and a finite sequence of neighbourhoods $U^{\varepsilon_1}_{x_1}, U^{\varepsilon_2}_{x_2}, \ldots, U^{\varepsilon_{n-1}}_{x_{n-1}}$ such that every $U^{\varepsilon_k}_{x_k}$ contains both ξ_k and ξ_{k-1}.

Let $\mathfrak{N}_\xi$ denote the totality of the elements $\zeta \in \mathfrak{M}$ which can be connected with ξ by finite chains. Evidently, if $\xi'' \in \mathfrak{N}_{\xi'}$ then $\xi' \in \mathfrak{N}_{\xi''}$.

Theorem 10. 1° $\mathfrak{N}_\xi$ *is both closed and open in* $\mathfrak{M}$; $\mathfrak{N}_\xi$ *cannot be represented as the sum of two non-overlapping closed sets. In other words,* $\mathfrak{N}_\xi$ *is a component of connectivity of the space* $\mathfrak{M}$. *Conversely, every component of connectivity* $\mathfrak{N}$ *of the space* $\mathfrak{M}$ *is* $\mathfrak{N}_\xi$ *for every* $\xi \in \mathfrak{N}$.

2°. $\mathfrak{N}$ *is a metrizable space.*

The Central Asia Un versity.
Tashkent.

Received
22. VI. 1944.

REFERENCES

[1] S. Banach, Théorie des opérations linéaires, 1932. [2] S. Lewin, Math. Z., 32, H. 4 (1930). [3] M. M. Grinblum, C. R. Acad. Sci. URSS, XXXI, No 5. (1941). [4] М. Крейн, М. Рутман и Д. Мильман, Зап. Харьковск. мат. общ., IV, сер. 16 (1940).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

MATHEMATICS

ON PURELY DISCONTINUOUS RANDOM PROCESSES WITH RESIDUAL EFFECT

By V. M. DUBROVSKY

(Communicated by A. N. Kolmogoroff, Member of the Academy, 8. VI. 1944)

Consider a system $\mathfrak{N}$ which may exist in states forming a set $\mathfrak{A}$. We assume that the transition from one state into another is instantaneous, and that the number of changes of the states is finite in any bounded segment of time, although it may be arbitrarily large. Let there be given a family $\mathfrak{M}$ of subsets of the space $\mathfrak{A}$ containing arbitrary differences and finite or enumerable sums of its elements, separate elements of $\mathfrak{A}$, the whole set $\mathfrak{A}$ and the void set. In the sequel we shall consider abstract Lebesgue—Stieltjes integrals and denote them by the symbol

$$\int_{\mathfrak{A}_k} f(x_k)\,\varphi(d\,\mathfrak{A}_k)$$

where the function $f\{x\}$ is measurable with respect to $\mathfrak{M}$ and bounded, the function of the set $\varphi(E)$ is totally additive ($x \subset \mathfrak{A}$, $E \subset \mathfrak{M}$); $\mathfrak{A}_k = \mathfrak{A}$, the index k showing that $\mathfrak{A}_k$ is the domain of integration corresponding to the variable $x_k (k = 1, 2, \ldots)$.

Let at the absolute initial moment t_0 the system $\mathfrak{N}$ be in the state x_0, and at the moment t, in the state x. Suppose that in the time interval $t_0 < s < t$ the changes of state occurred at the moments $t_1, t_2, \ldots, t_n$, and denote by x_k the stationary state in the time interval $t_k < s \leqslant t_{k+1}$ $(k = 1, 2, \ldots, n;$ $t_{n+1} = t,\ x_n = x;\ n = 0, 1, 2, \ldots)$.

Under these conditions let the probability of one change of state in the time $t \leqslant s < \tau$ be determined by the sum

$$p_n(t, x; t_1, x_1; t_2, x_2; \ldots; t_n, x_n)(\tau - t) + (\tau - t)\,\varepsilon_n$$

where ε_n is uniformly bounded and infinitely small, if $\tau - t$ tends to zero $(n = 0, 1, 2, \ldots)$. Let, under the additional condition that at the moment t occurs a change of state, the function $P_n(t, x, E; t_1, x_1; \ldots; t_n, x_n)$ express the probability that at the moments sufficiently near to t and subsequent to it, the state of the system will belong to the set $E (n = 0, 1, 2, \ldots)$. With respect to the functions p_n and P_n we assume that they are integrable in the sense of Riemann by every of the variables $t_1, t_2, \ldots$, non-negative and measurable with respect to $\mathfrak{M}$ in every of the variables $x_1, x_2, \ldots$. Besides, we assume that the functions p_n are uniformly bounded, and the functions P_n totally additive and do not exceed 1. All these assumptions are to be under-

stood as pertaining to the domain $-T < t_0 < t_1 < t_2 < \ldots < T$; $x_1 \subset \mathfrak{A}$, $x_2 \subset \mathfrak{A}$, $\ldots$; $E \subset \mathfrak{M}$, where T is a certain constant.

The processes of this kind in the case where the residual effect is absent were considered by Feller (1) and the author (2). The general problem is due to Kolmogoroff (3).

Denote by $\pi_n(t, x, \tau) = \pi_n(t, x, \tau; t_1, x_1; \ldots; t_n, x_n)$ the probability that there will be no change of state in the time $t \leqslant s < \tau$. The function $\pi_n(t, x, \tau)$ evidently satisfies the identity

$$\pi_n(t, x, \tau) = \pi_n(t, x, \tau+\Delta\tau) + \pi_n(t, x, \tau)\,[1 - \pi_n(\tau, x, \tau+\Delta\tau)]$$

On the other hand, denote by K the common upper bound of the functions p_n and ε_n, and we find

$$1 - \pi_n(\tau, x, \tau+\Delta\tau) = p_n(\tau, x; t_1, x_1; \ldots; t_n, x_n)\,\Delta\tau + \Delta\tau\,\varepsilon_n + \delta$$

where

$$\delta \leqslant (2K\Delta\tau)^2 + (2K\Delta\tau)^3 + \ldots$$

whence it follows that the function $\pi_n(t, x, \tau)$ is continuous in τ and satisfies the equation

$$\frac{\partial \pi_n(t, x, \tau)}{\partial \tau} = -\pi_n(t, x, \tau)\, p_n(\tau, x; t_1, x_1; \ldots; t_n, x_n)$$

where we have the right derivative, and consequently π_n is determined by the equality

$$\pi_n(t, x, \tau; t_1, x_1; \ldots; t_n, x_n) = e^{-\int_t^\tau p_n(s, x; t_1, x_1; \ldots; t_n, x_n)\, ds}$$

$$(n = 0, 1, 2, \ldots.)$$

Suppose it to be known that at the moment t the system is in some state from the set e ($e \subset \mathfrak{M}$). Consider the problem to define the probability $F(t, e, \tau, E)$ of the fact A, that is, the probability that at the moment τ the state will belong to the set E ($E \subset \mathfrak{M}$).

To solve this problem we make all possible hypotheses with respect to the course of the process up to the moment t, and determine their probabilities *a priori* as well as their probabilities *a posteriori* under the condition of realization of the fact that at the moment t the state belongs to the set e. To this end we apply the formulae of Bayes (which are formulated for the case of an infinite set of hypotheses in (4)). Then, forming the products of the probability of every hypothesis *a posteriori* by the probability of the fact A under the condition of this hypothesis, and summing up the obtained products, we obtain in the limit the required probability $F(t, e, \tau, E)$.

Putting

$$\varphi_{k+1}(E) = \pi_k(t_k, x_k, t_{k+1}; t_1, x_1; \ldots; t_k, x_k) \times$$

$$\times\, p_k(t_{k+1}, x_k; t_1, x_1; \ldots; t_k, x_k)\, P_k(t_{k+1}, x_k, E; t_1, x_1; \ldots; t_k, x_k)$$

where $k = 0, 1, 2, \ldots$ $\mathrm{E}(x, E) = 1$, if $x \subset E$, and $\mathrm{E}(x, E) = 0$, if $x \subset \mathfrak{A} - E$, it is easily seen that the probability $F(t, e, \tau, E)$ is determined in the following manner:

$$F(t, e, \tau, E) = \frac{1}{\sum_{n=0}^{\infty} Q_n} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} P(m, n)$$

where

$$P^{(m,\,n)} = \int_{t_0}^{t} dt_1 \int_{\mathfrak{A}_1} \cdots \int_{t_{n-1}} dt_n \int_{\mathfrak{A}_n} \int_{t}^{\tau} dt_{n+1} \int_{\mathfrak{A}_{n+1}} \int_{t_{n+1}}^{\tau} dt_{n+2} \int_{\mathfrak{A}_{n+2}} \cdots \int_{t_{n+m-1}}^{\tau} dt_{n+m} \int_{\mathfrak{A}_{n+m}} \Omega_{m,\,n};$$

$$\Omega_{m,\,n} = \mathrm{E}\,(x_n,\ e)\,\mathrm{E}\,(x_{n+m},\ E)\,\pi_{n+m}\,(t_{n+m},\ x_{n+m},\ \tau) \prod_{k=1}^{n+m} \varphi_k\,(d\mathfrak{A}_k);$$

$$Q_n = \int_{t_0}^{t} dt_1 \int_{\mathfrak{A}_1} \int_{t_1}^{t} dt_2 \int_{\mathfrak{A}_2} \cdots \int_{t_{n-1}}^{t} dt_n \int_{\mathfrak{A}_n} \mathrm{E}\,(x_n,\ e)\,\pi_n\,(t_n,\ x_n,\ t) \prod_{k=1}^{n} \varphi_k\,(d\mathfrak{A}_k)$$

It may be easily verified that the following estimations hold:

$$0 \leqslant P^{(m,\,n)} \leqslant \frac{(\tau - t)^m\,(t - t_0)^n}{m!\ n!}\,K^{m+n}$$

$$0 \leqslant Q_n \leqslant \frac{(t - t_0)^n}{n!}\,K^n$$

$(m,\ n = 0,\ 1,\ 2,\ \ldots)$, whence follows that the considered series converge absolutely and uniformly.

M. V. Lomonosov State University.
Moscow.

Received
8. VI. 1944.

REFERENCES

[1] W. Feller, Math. Ann., 113 (1936). [2] V. Dubrovsky, C. R. Acad. Sci. URSS, XIX, 439 (1938); Bull. Acad. Sci. URSS, sér. math., 4, 411 (1940). [3] A. Kolmogoroff, Math. Ann., 104 (1930). [4] А. Н. Колмогоров, Основные понятия теории вероятностей, М.—Л., 1936.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

MATHEMATICS

ON THE ANAMORPHOSIS OF SYMMETRIC EQUATIONS

By P. V. NIKOLAEV

(Communicated by A. N. Kolmogoroff, Member of the Academy, 27.V. 1944)

Consider the algebraic equation

$$F(t_1, t_2, t_3) \equiv \sum_{k=0}^{m_3} t_3^k \sum_{j=0}^{m_2} t_2^j \sum_{i=0}^{m_1} a_{jk}^{(i)} t_1^i = 0 \qquad (1)$$

symmetric in t_2 and t_3, *i. e.* an equation, the polynomial of which satisfies the identity

$$F(t_1, t_2, t_3) \equiv lF(t_1, t_3, t_2) \qquad (2)$$

where l is a certain constant. A double interchange of the variables t_2, and t_3 in the polynomial $F(t_1, t_2, t_3)$ leads to the identity

$$F(t_1, t_2, t_3) \equiv l^2 F(t_1, t_2, t_3) \qquad (2')$$

i. e. $l = \pm 1$.

If the canonic ([1]) decomposition of the polynomial $F(t_1, t_2, t_3)$ has the form

$$F(t_1, t_2, t_3) \equiv \sum_{k=1}^{r_3} f_{3k} \sum_{j=1}^{r_2} f_{2j} \sum_{i=1}^{r_1} \alpha_{jk}^{(i)} f_{1i} \qquad (3)$$

where $f_{1i} \equiv f_{1i}(t_i)$, then, in case the equation

$$f_{2j}(t) \equiv f_{3j}(t) \quad (j = 1, \ldots; r_2 = r_3) \qquad (4)$$

is symmetric with respect to t_2, t_3, the A-matrix $T^{(1)}$ of the polynomial $F(t_1, t_2, t_3)$ as well as its canonical A-matrix $\overline{T}^{(1)}$ will be symmetric for $l+1$ and skew-symmetric for $l=-1$.

In the sequel we shall assume that the equation (1) contains an unreducible factor depending on all three variables and is free from factors depending only on one of the variables.

If the equation (1) admits of a rational anamorphosis and among its nomograms there is none of genus zero with respect to t_1, then the bases of the variables t_2 and t_3 in its anamorphoses will coincide and, besides, the scale equations of these variables will be identical. Hence follows that for the equations of the described type there exists such an anamorphosis in which the elements of the lines of the variables t_2 and t_3 are the generatrices of the

canonic decomposition of the (reduced) Massau polynomial $\Phi(t_1, t_2, t_3)$ corresponding to the equation (1). Such an anamorphosis we shall call canonic.

The polynomial $F(t_1\ t_2,\ t_3)$ of the equation of the form

$$F(t_1\ t_2,\ t_3) \equiv \sum_{k=1}^{r} f_{3k} \sum_{j=1}^{r} f_{2j} \sum_{i=1}^{3} \alpha_{jk}^{(i)} f_{1i} = 0 \qquad (1')$$

symmetric in t_2, t_3, is then and only then the Massau polynomial, if in its canonic decomposition (1') $r=3$ and its A-matrix $T^{(1)}$ is skew-symmetric.

The canonic A-matrix $T^{(1)}$ for the polynomials (1') will, under the conditions stated, have the form

$T^{(1)}$	f_{31}	f_{32}	f_{33}
f_{21}	0	$\sum \alpha_{12}^{(i)} f_{1i}$	$-\sum \alpha_{13}^{(i)} f_{1i}$
f_{22}	$-\sum \alpha_{12}^{(i)} f_{1i}$	0	$\sum \alpha_{23}^{(i)} f_{1i}$
f_{23}	$\sum \alpha_{13}^{(i)} f_{1i}$	$-\sum \alpha_{23}^{(i)} f_{1i}$	0

(5)

i. e. will be the Massau matrix. Consequently, these polynomials possess a canonic anamorphosis.

As regards the equations of the form

$$F(t_1,\ t_2,\ t_3) \equiv \sum_{k=1}^{r} f_{3k} \sum_{j=1}^{r} f_{3j} \sum_{i=1}^{2} \alpha_{jk}^{(i)} f_{1i} = 0 \qquad (1'')$$

they may be anamorphosized also in the case where there is no canonic anamorphosis; as an example may serve, beside the equations of genus zero, the equation

$$f_{11}(f_{21}+f_{31}) - f_{12}(f_{22}+f_{32}) = 0, \quad [f_{2j}(t) \equiv f_{3j}(t)] \qquad (6)$$

symmetric in t_2, t_3.

In order that the polynomial of the equation (1'') should be the Massau polynomial with a canonic anamorphosis, it is necessary and sufficient that $r=3$ and that its A-matrix $T^{(1)}$ should be skew-symmetric.

We shall call the algebraic equation $F(t_1,\ t_2,\ t_3)=0$ (the polynomial $F(t_1,\ t_2,\ t_3)$) an equation (polynomial) of the type $[r_1,\ r_2,\ r_3]$, if r_i is the number of terms in the canonic by t_i decomposition of the polynomial $F(t_1,\ t_2,\ t_3)$. As we stated already, the equations of the types [2; 3; 3], [3; 2; 2] and [3; 3; 3] always admit of a direct anamorphosis, if the A-matrix $T^{(i)}$ of the polynomial $F(t_1,\ t_2,\ t_3)$ is skew-symmetric; the same will hold, as the example (6) shows, for certain equations of the type [2; 3; 3] (and only this type) also in the case of a symmetric A-matrix.

Consider the question on the anamorphosibility of normal, symmetric with respect to t_2, t_3, equations (1) by means of a factor $\psi_1(t_2,\ t_3)$. If the equation (1) admits of an anamorphosizing factor $\psi_1(t_2,\ t_3)$ and $\Phi(t_1,\ t_2,\ t_3) \equiv \psi_1(t_2,\ t_3) F(t_1,\ t_2,\ t_3)$ is, consequently, a Massau polynomial, then the number of terms in every canonic decomposition of the polynomial $\Phi(t_1,\ t_2,\ t_3)$ will be equal to 2 or 3. But the systems of polynomials $\Phi(t_1,\ t_2,\ t_3)$ and $F(t_1,\ t_2,\ t_3)$ generating the decompositions by t_1 coincide. Hence follows that this number will be the same also for $F(t_1,\ t_2,\ t_3)$. Besides, the polynomial $F(t_1,\ t_2,\ t_3)$ must be symmetric with respect to t_2, t_3, since in the case of a skew-symmetric A-matrix $T^{(1)}$ of the polynomial $F(t_1,\ t_2,\ t_3)$ it has the factor $(t_2 - t_3)$

and cannot therefore admit of normal anamorphoses by means of a factor $\psi(t_2, t_3)$ [2].

As regards the form of the anamorphosizing factor in the case of a normal equation symmetric in t_2, t_3, for the equations of the type $[3; r; r]$ $\psi_1(t_2, t_3) = (t_2 - t_3)$ (up to a constant factor), and for the equations of the type $[2; r; r]$ of genus different from zero, beside the indicated form is also possible the form

$$\psi_1(t_2, t_3) \equiv a t_2 t_3 + b(t_2 + t_3) + d \tag{7}$$

where a, b, d are certain constants. This follows from the fact that in the case of normal anamorphoses $\psi_1(t_2, t_3)$ is a linear function of t_2 and of t_3, and $\Phi(t_1, t_2, t_3) = 0$ is an equation symmetric with respect to these variables; for the equation of the type $[3; r; r]$ $\Phi(t_1, t_2, t_3)$ possesses, besides, only a skew-symmetric A-matrix $T^{(1)}$.

We shall state the conditions under which the equation (1) admits of an anamorphosizing factor $(t_2 - t_3)$. Write the equation (1) in the form

$$F(t_1, t_2, t_3) \equiv \sum_{k=0}^{m} t_3^k \sum_{j=0}^{m} t_2^j \sum_{i=1}^{r_1} a_{jk}^{(i)} f_{1i} \equiv \sum_{k=0}^{m} t_3^k F_k^{(12)} = 0. \tag{8}$$

If (8) admits of a factor $(t_2 - t_3)$, then it is necessary that for $r_1 = 3$ the rank of the A-matrix (with constant elements) of the polynomial $F_m^{(12)} \equiv \overline{F}_m(t_1, t_2)$ should be equal to 2. This follows directly from the fact that $F_m^{(12)} \equiv -\overline{F}_1^{(12)}$, where $F_1^{(12)}$ is the coefficient of the canonic in t_3 decomposition of the Massau polynomial

$$\Phi(t_1, t_2, t_3) \equiv (t_2 - t_3) F(t_1, t_2, t_3) \equiv \sum_{k=1}^{3} \overline{f}_{3k} \overline{F}_k^{(12)} \tag{9}$$

and that these coefficients are the algebraic complements in the Massau determinant

$$\Phi(t_1, t_2, t_3) \equiv |\overline{f}_{l_1}, f_{l_2}, f_{l_3}| \tag{10}$$

with linearly independent elements.

The degrees of the polynomials f_{l2}, f_{l_3} $(l = 2, 3)$ in the canonical anamorphosis (10) we shall call the characteristic numbers of the equation (8).

For $r_1 = 3$ these numbers may be determined from the A-matrix of the polynomial $F_m^{(12)}$

$\overline{T}^{(3)}$	t_2^m ..	t_2^μ ..	t_2^ν ..	1
f_{11}	$a_{mm}^{(1)}$ ..	$a_{\mu m}^{(1)}$ ..	$a_{\nu m}^{(1)}$ ..	$a_{0m}^{(1)}$
f_{12}	$a_{mm}^{(2)}$ ..	$a_{\mu m}^{(2)}$ ..	$a_{\nu m}^{(2)}$ ..	$a_{0m}^{(2)}$
f_{13}	$a_{mm}^{(3)}$ ..	$a_{\mu m}^{(3)}$ ..	$a_{\nu m}^{(3)}$ ..	$a_{0m}^{(3)}$

(11)

Namely, if the columns $a_{\mu m}^{(i)}$ and $a_{\nu m}^{(i)}$ $(i = 1, 2, 3)$ are the first which cannot be linearly expressed through the preceding ones, the μ and ν are the characteristic numbers of the equation (8) admitting of the factor $(t_2 - t_3)$.

For $r_1 = 2$ the matrix (11) determines in the case of linear dependence of the elements f_{12} and f_{13} only one of these numbers. Suppose, for instance, that it is the number μ. Then the second number ν may be determined from the A-matrix of the polynomial $-F_{\mu-1}^{(12)} + t_2 F_\mu^{(12)}$ of the form (11).

If now μ and ν are the characteristic numbers of the equation (8), then the coefficients of the canonic in t_3 decomposition of the polynomial (9) $\Phi(t_1, t_2, t_3)$ will be

$$\overline{F}_1^{(12)} = -F_m^{(12)}, \quad \overline{F}_2^{(12)} = -\overline{F}_{\mu-1}^{(12)} + t_2 F_\mu^{(12)}, \quad \overline{F}_3^{(12)} = -F_{\nu-1}^{(12)} + t_2 F_\nu^{(12)} \tag{12}$$

Hence we obtain the following necessary conditions for equation $F(t_1, t_2, t_3) = 0$ to admit of an anamorphosizing factor $(t_2 - t_3)$:

a) $F(t_1, t_2, t_3)$ is a polynomial symmetric with respect to t_2, t_3;

b) in the sequence

$$-F_m^{(12)};\ -F_{m-1}^{(12)} + t_2 F_m^{(12)};\ \ldots;\ -F_{\mu-1}^{(12)} + t_2 F_\mu^{(12)};\ \ldots;\ -F_{\nu-1}^{(12)} + t_2 F_\nu^{(12)} \tag{13}$$

there are three terms

$$F_1^{(12)},\ F_2^{(12)},\ F_3^{(12)} \tag{12'}$$

such that every term of the sequence (13) satisfies identically the equality

$$-F_{j-1}^{(12)} + t_2 F_j^{(12)} \equiv \beta_j \overline{F}_1^{(12)} + \gamma_j \overline{F}_2^{(12)} + \delta_j \overline{F}_3^{(12)} \quad (j = 0, \ldots, m+1) \tag{14}$$

where β_j, γ_j, δ_j are certain constants.

The conditions a) and b) are also sufficient. If they are satisfied, the elements of the canonic anamorphosis (10) are given by the formulae

$$\left.\begin{aligned}
&\overline{f}_{l1} = t_l^{m+1} + \sum_{k=0}^{m} \beta_k t_l^k, \qquad \overline{f}_{11} = \sum_{i=1}^{3} \left[a_{\nu,\mu-1}^{(i)} - a_{\nu-1,\mu}^{(i)} \right] f_{1i} \\
&\overline{f}_{l2} = t_l^{\mu} + \sum_{k=0}^{\mu-1} \gamma_k t_l^k, \qquad \overline{f}_{12} = -\sum_{i=1}^{3} a_{\nu m}^{(i)} f_{1i} \\
&\overline{f}_{l3} = t_l^{\nu} + \sum_{k=0}^{\nu-1} \delta_k t_l^k, \qquad \overline{f}_{13} = \sum_{i=1}^{3} a_{\mu m}^{(i)} f_{1i} \\
&\qquad (l = 2,3;\ \beta_\mu = \beta_\nu = \gamma_\nu = 0).
\end{aligned}\right\} \tag{15}$$

Consider now the tertiary symmetric equation

$$F(t_1, t_2, t_3) \equiv \sum_{k=1}^{r} f_{3k} \sum_{i=1}^{r} f_{2i} \sum_{i=1}^{r} a_{jk}^{(i)} f_{1i} = 0 \quad \left[f_{1l}(t) \equiv f_{2l}(t) \equiv f_{3l}(t) \right] \tag{16}$$

i. e. an equation symmetric with respect to any pair of the variables t_1, t_2, t_3.

All A-matrices of the polynomial $F(t_1, t_2, t_3)$ of the equation (16) are simultaneously symmetric and skew-symmetric, since from the identities

$$F(t_1, t_2, t_3) \equiv l_1 F(t_1, t_3, t_2) \equiv l_2 F(t_3, t_2, t_1) \equiv l_3 F(t_2, t_1, t_3) \tag{17}$$

follows that $l_1 = l_2 = l_3 = \pm 1$.

For $r = 2$ the question as to the anamorphoses of the equation (16) is clear. In other cases the polynomial $F(t_1, t_2, t_3)$ will be then and only then the Massau polynomial, if its A-matrices are skew-symmetric and its canonic decompositions have each three terms $(r = 3)$.

As regards the anamorphosizing factor for $r \geqslant 3$, in the case of normal anamorphoses it exists only for symmetric (tertiary) polynomials $F(t_1, t_2, t_3)$, *i. e.* polynomials of the equation (16) with symmetric A-matrices. Hereby the anamorphosizing factor has the unique form

$$\psi_1(t_1, t_2, t_3) \equiv (t_1 - t_2)(t_2 - t_3)(t_3 - t_1). \tag{18}$$

In order that the equation

$$F(t_1,t_2,t_3)\equiv\sum_{k=0}^{m}t_3^kF_3^{(12)}=0 \tag{19}$$

should admit of an anamorphosizing factor of the form (18), it is necessary and sufficient that the following conditions should be satisfied:

a) $F(t_1, t_2, t_3)$ is a symmetric (tertiary) polynomial;

b) there are in the sequence

$$-\overline{F}_{j-1}^{(12)}+(t_1+t_2)F_j^{(12)}-t_1t_2F_{j+1}^{(12)}\quad(j=0,\ldots,m+1) \tag{20}$$

three terms $\overline{F}_1^{(12)}, \overline{F}_2^{(12)}, \overline{F}_3^{(12)}$, such that every term of the sequence (20) is a linear combination of three terms.

These conditions may be established, if we apply to the equation

$$'F(t_1, t_2, t_3)\equiv(t_1-t_2)(t_3-t_1)F(t_1, t_2, t_3)=0 \tag{21}$$

the necessary and sufficient conditions for the existence of the anamorphosing factor (t_2-t_3).

From the sequence (13) it is easy to obtain the sequence (20); the elements of the canonic anamorphosis of the equation (19) may be found by means of formulae (14) and (15).

Received
9. V. 1944.

REFERENCES

[1] P. V. Nikolaev, C. R. Acad. Sci. URSS, XXVIII, No. 9 (1940). [2] Idem, ibid., XXVIII, No. 7 (1940).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

THEORY OF ELASTICITY

FUNDAMENTAL EQUATIONS OF THE THEORY OF SHELLS

By J. N. RABOTNOV

(Communicated by L. S. Leibenson, Member of the Academy, 31. V. 1944)

§ 1. Stresses and Strains of an Infinitely Thin Layer. Let us consider an infinitely thin layer with reference to curvilinear coordinates u, v. Upon deformation the coefficients g_{ij} receive increments $2\varepsilon_{ij}$, and the relative elongation of an element will be

$$\varepsilon = \frac{\varepsilon_{11}du^2 + 2\varepsilon_{12}du\,dv + \varepsilon_{22}dv^2}{ds^2} \tag{1.1}$$

The sum of main elongations is

$$\varepsilon_1 + \varepsilon_2 = \vartheta = \frac{g_{11}\varepsilon_{22} + g_{22}\varepsilon_{11} - 2g_{12}\varepsilon_{12}}{g} \tag{1.2}$$

where $g = g_{11}g_{22} - g_{12}^2$.

If for an infinitely small triangle in equilibrium the stresses applied to its sides, formed by coordinate lines, be resolved along the tangents to those lines, we will find that the normal stress upon an element ds is

$$\sigma = \frac{s_{11}du^2 + 2s_{12}du\,dv + s_{22}dv^2}{ds^2} \tag{1.3}$$

Here $s_{11} = \sigma_{22}\sqrt{g_{11}g/g_{22}}$, $s_{12} = \sigma_{12}\sqrt{g}$, $s_{22} = \sigma_{11}\sqrt{g_{22}g/g_{11}}$.

The vector of the stress applied to ds may be written down as

$$\bar{\sigma}\,ds = \bar{s}_1 du + \bar{s}_2 dv \tag{1.4}$$

Here

$$\bar{s}_1 = \frac{1}{\sqrt{g}}\left(s_{11}\bar{\rho}_2 - s_{12}\bar{\rho}_1\right), \quad \bar{s}_2 = \frac{1}{\sqrt{g}}\left(s_{12}\bar{\rho}_2 - s_{22}\bar{\rho}_1\right) \tag{1.5}$$

The relation between the stresses and strains is described by the equations

$$s_{ij} = \frac{E}{1-\mu^2}\left[\vartheta\, g_{ij} - (1-\mu)\,\varepsilon_{ij}\right] \tag{1.6}$$

§ 2. The State of a Shell under Stress and Strain. A shell of finite thickness $2h$ whose middle surface is determined by the radius-vector $\bar{\rho}$ given as a function of two parameters u, v, can be imagined to be composed of infinitely thin layers parallel to the middle surface. The stress- and strain state of one layer is described by the formulae of the preceding paragraph.

The radius-vector of a point of the layer at a distance z from the middle surface

$$\bar{\rho}' = \bar{\rho} + \bar{\nu}z \tag{2.1}$$

Here $\bar{\nu}$ is the unit vector of the normal to the middle surface. From the relation (2.1) g'_{ij} can be computed for the layer. We have

$$g'_{ij} = g_{ij} - 2b_{ij}z \tag{2.2}$$

$b_{ij} = -\bar{\rho}_i \bar{\nu}_j$ are the coefficients of the second quadratic form for the mid-layer. Upon deformation b_{ij} changes by as much as δ_{ij} and, if Kirchhoff's hypothesis remains valid, the strain components of the layer are determined as follows:

$$\varepsilon'_{ij} = \varepsilon_{ij} - \delta_{ij}z \tag{2.3}$$

Six quantities $\varepsilon_{ij}, \delta_{ij}$ completely determine the strained state of the shell. Its stress state is found by means of formulae (1.6). Disregarding z^2, we obtain

$$s'_{ij} = s_{ij} - zp_{ij}$$

the s_{ij} are determined from (1.6), and for the p_{ij} we have

$$p_{ij} = \frac{E}{1-\mu^2}\left[\delta g_{ij} - (1-\mu)\delta_{ij}\right], \quad \delta = \frac{1}{g}\left(g_{12}\delta_{22} + g_{22}\delta_{11} - 2g_{12}\delta_{12}\right) \tag{2.4}$$

In order that the stress state of the shell as well as the strains be characterized by co-variant parameters, we will take the following six quantities for the measure of internal forces:

$$T_{ij} = \int_{-h}^{+h} s'_{ij}\,dz = 2hs_{ij}, \quad L_{ij} = \int_{-h}^{+h} zs'_{ij}\,dz = \frac{2h^3}{3}p_{ij} \tag{2.5}$$

The expressions

$$T = \frac{T_{11}\,du^2 + 2T_{12}\,du\,dv + T_{22}\,dv^2}{ds^2} \quad \text{and} \quad L = \frac{L_{11}du^2 + 2L_{12}\,du\,dv + L_{22}\,dv^2}{ds^2}$$

differ from the normal stress and the bending moment by a quantity of order h/R if the bending stresses and those of tension and compression are magnitudes of the same order.

For T_{ij}, L_{ij} we obtain the following formulae:

$$T_{ij} = C\left[g_{ij}\vartheta - (1-\mu)\varepsilon_{ij}\right], \quad L_{ij} = -B\left[g_{ij}\vartheta - (1-\mu)\delta_{ij}\right] \tag{2.6}$$

§ 3. Equilibrium Equations. If R_1 is the shearing force per a unit length of the line u, and R_2 is the shearing force per a unit length of the line v, we shall find from the conditions of equilibrium of an infinitely small triangle that

$$Rds = R_1\sqrt{g_{11}}\,du + R_2\sqrt{g_{22}}\,dv = N_1\,du + N_2\,dv \tag{3.1}$$

Here $R_i\sqrt{g_{ii}} = N_i$.

The force acting on a unit area of the mid-surface is

$$\bar{q} = q_1\bar{\rho}_1 + q_2\bar{\rho}_2 + q_n\bar{\nu}$$

Let us add some new symbols to the notations adopted in § 1:

$$\sqrt{g}\,\bar{T}_i = T_{i1}\bar{\rho}_2 - T_{i2}\bar{\rho}_1, \qquad \sqrt{g}L_i = L_{i1}\bar{\rho}_2 - L_{i2}\bar{\rho}_1$$
$$\sqrt{g}\,\bar{\tau}_i = T_{i1}\bar{\nu}_2 - T_{i2}\bar{\nu}_1, \qquad \sqrt{g}\lambda_i = L_{i1}\bar{\nu}_2 - L_{i2}\bar{\nu}_1$$

We are going to consider the equilibrium state of the shell's portion cut out by the surface formed by the normals to a contour Γ enclosing an area F

on the middle surface. Putting at zero the main vector of the system of acting forces we get

$$\int_{-h}^{+h} dz \int_{\Gamma} (\bar{s}_1' du + \bar{s}_2' dv) + \int_{\Gamma} \bar{\nu} (N_1 du + N_2 dv) + \iint_F \bar{q} g \, du \, dv = 0 \qquad (3.2)$$

The first term of the integrand is:

$$\frac{s_{11}' \bar{\rho}_2'}{\sqrt{g'}} = (s_{11} - z p_{11}) (\bar{\rho}_2 + z \bar{\nu}_2)(1 - zH) \frac{4}{\sqrt{g}} =$$
$$= \frac{s_{11} \bar{\rho}_2}{\sqrt{g_2}} - \frac{z^2}{\sqrt{g}} (p_{11} \bar{\nu}_2 + s_{11} \bar{\nu}_2 H - p_{11} \bar{\rho}_2 H) + \ldots$$

(Terms with the odd powers of z are omitted, H is the mean curvature). After the other terms of the integrand are treated similarly and the contour integrals are transformed into surface integrals, we get

$$\left(\frac{\partial \bar{T}_1}{\partial v} - \frac{\partial \bar{T}_2}{\partial u}\right) + \left(\frac{\partial \bar{\lambda}_1}{\partial v} - \frac{\partial \bar{\lambda}_2}{\partial u}\right) - \frac{h^2}{3} \left[\frac{\partial}{\partial v} (H \tau_1) - \frac{\partial}{\partial u} (H \bar{\tau}_2)\right] -$$
$$- \left[\frac{\partial}{\partial v} (H \bar{L}_1) - \frac{\partial}{\partial u} (H \bar{L}_2)\right] + \bar{\nu} \left(\frac{\partial N_1}{\partial v} - \frac{\partial N_2}{\partial u}\right) + (N_1 \bar{\nu}_2 - N_2 \bar{\nu}_1) + qg = 0 \qquad (3.3)$$

Let us construct the condition under which the main moment

$$\int_{-h}^{+h} dz \int_{\Gamma} (\bar{s}_1' \times \bar{\rho}' \, du + \bar{s}_2' \times \bar{\rho}' \, dv) + \int_{\Gamma} (N_1 du + N_2 dv) \bar{\nu} \times \bar{\rho} +$$
$$+ \iint_F \bar{q} \times \rho g \, du dv \qquad (3.4)$$

is equal to zero. In order to transform the first integral we will consider the first term of the integrand:

$$\frac{s_{11}' \rho_2' \times \rho}{\sqrt{g'}} = (s_{11} - z p_{11}) (1 - zH) (\bar{\rho}_2 + z \bar{\nu}_2) \times (\bar{\rho} + z \bar{\nu}) \frac{1}{\sqrt{g}} =$$
$$= \frac{s_{11} \bar{\rho}_2 \times \bar{\rho}}{\sqrt{g}} - \frac{z^2}{\sqrt{g}} (p_{11} \bar{\nu}_2 \times \bar{\rho} - s_{11} H \bar{\nu}_2 \times \bar{\rho} - p_{1,} H \bar{\rho}_2 \times \bar{\rho} + p_{11} \bar{\rho}_2 \times$$
$$\times \bar{\nu} + s_{11} \bar{H} \rho_2 \times \bar{\nu} - s_{11} \bar{\nu}_2 \times \bar{\nu}) + \ldots$$

Computing similarly the second and other terms and integrating with respect to z we get:

$$\int_{-h}^{+h} \bar{s}_1' \times \bar{\rho}' dz = \left(\bar{T}_1 + \bar{\lambda}_1 - \frac{h^2}{3} H \bar{\tau}_1 - H \bar{L}_1\right) \times \bar{\rho} + \left(\bar{L}_1 - \frac{h^2}{3} H \bar{T}_1 + \frac{h^2}{3} \bar{\tau}_1\right) \times \bar{\nu}$$

Transforming the contour integrals of equation (3.4) into surface integrals we find that

$$\left\{\left(\frac{\partial \bar{L}_1}{\partial v} - \frac{\partial \bar{L}_2}{\partial u}\right) - \frac{h^2}{3} \left(\frac{\partial}{\partial v} (H \bar{T}_1) - \frac{\partial}{\partial u} (H \bar{T}_2)\right) + \frac{h^2}{3} \left(\frac{\partial \bar{\tau}_1}{\partial v} - \frac{\partial \bar{\tau}_2}{\partial u}\right) -\right.$$
$$\left. - (N_1 \bar{\rho}_2 - N_2 \bar{\rho}_1)\right\} \times \bar{\nu} = 0 \qquad (3.5)$$

In simplifying use was made of equation (3.3) and of the identities

$$T_1 \times \bar{\rho}_2 - \bar{T}_2 \times \bar{\rho}_1 = 0, \qquad \bar{\tau}_1 \times \bar{\nu}_2 - \bar{\tau}_2 \times \bar{\nu}_1 = 0,$$
$$\bar{\tau}_1 \times \bar{\rho}_2 - \bar{\tau}_2 \times \bar{\rho}_1 = -(\bar{T}_1 \times \bar{\nu}_2 - T_2 \times \bar{\nu}_1)$$

and of the respective identities for $\bar{L}_i$ and $\bar{\lambda}_i$.

Assuming that the bending and tensile stresses of the middle surface are magnitudes of the same order, and neglecting the quantity h/R in comparison with unity we shall find that the equations (3.3) and (3.5) admit of simplification. Namely

$$\left(\frac{\partial \overline{T_1}}{\partial v}-\frac{\partial \overline{T_2}}{\partial u}\right)+\bar{\nu}\left(\frac{\partial N_1}{\partial v}-\frac{\partial N_2}{\partial u}\right)+(N_1\bar{\nu}_2-N_2\bar{\nu}_1)+\bar{q}g=0$$

$$\left\{\left(\frac{\partial \overline{L_1}}{\partial v}-\frac{\partial \overline{L_2}}{\partial u}\right)-(N_1\bar{\rho}_2+N_2\bar{\rho}_1)\right\}\times\bar{\nu}=0$$

In the scalar form:

$$\frac{\partial T_{11}}{\partial v}-\frac{\partial T_{12}}{\partial u}-\left\{\begin{matrix}12\\1\end{matrix}\right\}T_{11}+\left(\left\{\begin{matrix}11\\2\end{matrix}\right\}-\left\{\begin{matrix}12\\2\end{matrix}\right\}\right)T_{12}+\left\{\begin{matrix}11\\2\end{matrix}\right\}T_{22}-$$
$$-\frac{g_{12}b_{12}-g_{11}b_{22}}{g}N_1+\frac{g_{12}b_{12}-g_{11}b_{12}}{g}N_2+q_1g=0,$$

$$\frac{\partial T_{22}}{\partial u}-\frac{\partial T_{12}}{\partial v}+\left\{\begin{matrix}22\\1\end{matrix}\right\}T_{11}+\left(\left\{\begin{matrix}22\\2\end{matrix}\right\}-\left\{\begin{matrix}12\\1\end{matrix}\right\}\right)T_{12}-\left\{\begin{matrix}12\\2\end{matrix}\right\}T_{22}+$$
$$+\frac{g_{12}b_{22}-g_{22}b_{12}}{g}N_1-\frac{g_{12}b_{12}-g_{11}b_{11}}{g}N_2+q_2g=0,$$

$$T_{11}b_{22}+T_{22}b_{11}-2T_{12}b_{12}+g\left(\frac{\partial N_1}{\partial v}-\frac{\partial N_2}{\partial u}+q_n\right)=0,$$

$$\frac{\partial L_{11}}{\partial v}-\frac{\partial L_{22}}{\partial u}-\left\{\begin{matrix}12\\1\end{matrix}\right\}L_{11}+\left(\left\{\begin{matrix}11\\1\end{matrix}\right\}-\left\{\begin{matrix}12\\2\end{matrix}\right\}\right)L_{12}+\left\{\begin{matrix}11\\2\end{matrix}\right\}L_{22}-N_1g=0,$$

$$\frac{\partial L_{22}}{\partial u}-\frac{\partial L_{12}}{\partial v}+\left\{\begin{matrix}22\\1\end{matrix}\right\}L_{11}+\left(\left\{\begin{matrix}22\\2\end{matrix}\right\}-\left\{\begin{matrix}12\\1\end{matrix}\right\}\right)L_{12}-\left\{\begin{matrix}12\\2\end{matrix}\right\}L_{22}+N_2g=0$$

§ 4. Equations of Compatibility. The components of the deformation and of the change of curvature are connected with each other by the relations which follow from Codazzi and Gauss' formulae. As a result of the deformation of the mid-surface the values of the Christoffel symbols undergo a change. Namely

$$\left\{\begin{matrix}ij\\k\end{matrix}\right\}'=\left\{\begin{matrix}ij\\k\end{matrix}\right\}+\gamma_{ij,k}-2\left\{\begin{matrix}ij\\k\end{matrix}\right\}\vartheta$$

The quantities $\gamma_{ij,k}$ are computed directly. For instance

$$\gamma_{11,2}=\frac{1}{g}\left[-g_{12}\frac{\partial\varepsilon_{11}}{\partial u}+2g_{11}\frac{\partial\varepsilon}{\partial v}-g_{11}\frac{\partial\varepsilon_{11}}{\partial v}-\varepsilon_{12}\frac{\partial g_{11}}{\partial u}+2\varepsilon_{11}\frac{\partial g_{12}}{\partial u}-\varepsilon_{11}\frac{\partial g_{11}}{\partial v}\right],$$

$$\gamma_{12,2}=\frac{1}{g}\left[g_{11}\frac{\partial\varepsilon_{22}}{\partial u}-g_{12}\frac{\partial\varepsilon_{11}}{\partial v}+\varepsilon_{11}\frac{\partial g_{22}}{\partial u}-\varepsilon_{12}\frac{\partial g_{11}}{\partial u}\right] \qquad (4.1)$$

If the deformations of the extreme fibre due to tension or compression and those due to bending are magnitudes of the same order, the terms with $\gamma_{ij,k}$ can be neglected. We then get

$$\frac{\partial\delta_{11}}{\partial v}-\frac{\partial\delta_{12}}{\partial u}-\left\{\begin{matrix}12\\1\end{matrix}\right\}\delta_{11}+\left(\left\{\begin{matrix}11\\1\end{matrix}\right\}-\left\{\begin{matrix}12\\2\end{matrix}\right\}\right)\delta_{12}+\left\{\begin{matrix}11\\2\end{matrix}\right\}\delta_{22}=0$$

$$\frac{\partial\delta_{22}}{\partial u}-\frac{\partial\delta_{12}}{\partial v}+\left\{\begin{matrix}22\\1\end{matrix}\right\}\delta_{11}+\left(\left\{\begin{matrix}22\\2\end{matrix}\right\}-\left\{\begin{matrix}12\\1\end{matrix}\right\}\right)\delta_{12}-\left\{\begin{matrix}12\\2\end{matrix}\right\}\delta_{22}=0 \qquad (4.2)$$

The third equation can be written in different forms, taking different forms of Gauss' relation for starting point. One of these forms is as follows:

$$b_{11}\delta_{22}+b_{22}\delta_{11}-2b_{12}\delta_{12}=\sqrt{g}\left[\frac{\partial}{\partial v}\left(\frac{\sqrt{g}}{g_{11}}\gamma_{11,2}\right)-\frac{\partial}{\partial u}\left(\frac{\sqrt{g}}{g_{11}}\gamma_{12,2}\right)\right]-$$
$$-\frac{\sqrt{g}}{g_{11}}\left[\left\{\begin{matrix}11\\2\end{matrix}\right\}\frac{\partial}{\partial v}\left(\vartheta+2\frac{\varepsilon_{11}}{g_{11}}\right)-\left\{\begin{matrix}12\\2\end{matrix}\right\}\frac{\partial}{\partial u}\left(\vartheta+2\frac{\varepsilon_{11}}{g_{11}}\right)\right]-2(b_{11}b_{22}-b_{12}^2)\frac{\varepsilon_{11}}{g_{11}} \qquad (4.3)$$

Received
31. V. 1944.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

HYDRODYNAMICS

ON UNSTEADY MOTIONS OF A COMPRESSIBLE FLUID

By L. I. SEDOV

(*Communicated by N. E. Kochin, Member of the Academy, 14. VI. 1944*)

Some considerations connected with the theory of dimension have led the author to develop a general method for finding exact solutions of physical equations. This method is simple, and in a number of cases families of solutions depending on several arbitrary parameters can be obtained by means of it. Generally, the physical hypotheses which lead to the solutions of this type are easy to characterize. Just what this method consists in, will be made clear in this paper where it will be employed to obtain new important families of exact solutions for the equations of unsteady one-dimensional motions of a compressible fluid.

One may point to a number of cases where the theory of dimension has been appealed to in order to improve some results already known before. But new solutions of the equations of mathematical physics do not appear to have ever been obtained on the basis of the theory of dimension. Nor has this theory been used as yet for elucidating the properties of the families of solutions that can be obtained in this way.

Considering one-dimensional insteady motions of a fluid, let us take the time t and a linear coordinate r for independent variables. The main quantities to be found are the velocity of the particles of the fluid $v\,([v]=LT^{-1})$, density $\rho\,([\rho]=ML^{-3})$ and pressure $p\,([p]=ML^{-1}T^{-2})$. It will be obvious that the functions $\rho(r, t)$ and $p(r, t)$ necessarilly contain some dimensional physical constants, since the dimensions of both density and pressure depend upon the symbol describing the unit of mass.

Let us examine the case where the functions sought for contain dimensional constants among which only one or two have independent dimensions. These special motions may have an independent interest or else they may prove to be limit motions, if among the given physical characteristics only one or two, having independent dimensions, are relevant in they limit.

Let us first consider the case where we have only one dimensional constant, which we denote by a. Without restraining the generality, suppose that the formula of dimension for a has the form

$$[a]=ML^kT^s$$

where k and s are constants. For v, ρ and p we can write

$$\left.\begin{aligned} v &= \frac{r}{t}V \\ \rho &= \frac{aR}{r^{k+3}t^s} \\ p &= \frac{aP}{r^{k+1}t^{s+2}} \end{aligned}\right\} \qquad (1)$$

The abstract quantities V, R and P cannot possibly depend upon r and t, and so they are of necessity constant. Thus, the hypothesis that the constant a alone is relevant affords the possibility of completely defining the dependence of v, ρ and p upon r and t.

If besides the constant a there is another constant b

$$[b] = L^m T^n$$

where m and n are constant*, then a variable abstract parameter

$$\lambda = b r^{-m} t^{-n}$$

can be formed from the determining quantities r, t, a and b.

In this case V, R and P may possibly depend upon λ. The assumption that only two constants a and b are relevant permits the distribution of values of v, ρ and p for $t=0$ to be established in a general form. From the theory of dimensionability we find:

$$\left.\begin{aligned} v &= v_0 b^{-\frac{1}{n}} r^{1+\frac{m}{n}} \\ \rho &= \rho_0 a b^{-\frac{s}{n}} r^{\frac{ms}{n}-k-3} \\ p &= p_0 a b^{-\frac{s}{n}-\frac{2}{n}} r^{\frac{ms}{n}-k-1+\frac{2m}{n}} \end{aligned}\right\} \qquad (2)$$

where v_0, ρ_0 and p_0 are abstract constants which may take different values including zero and infinity**.

In order to obtain the conditions which the quantities entering into (1) should satisfy, one has to turn to the equations of motion. It will be obvious that for the adopted hypotheses to be true, it is necessary in the general case that the equations of motion contain no other dimensional quantities besides those entering into formulae (1).

In order to render the problem definite let us have the equations of motion of the fluid in the following form:

$$\left.\begin{aligned} &\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 \\ &\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial r} + \frac{(\nu-1)\rho v}{r} = 0 \end{aligned}\right\} \qquad (3)$$

$$\frac{\partial \frac{p}{\rho^\gamma}}{\partial t} + v\frac{\partial \frac{p}{\rho^\gamma}}{\partial r} = 0 \qquad (4)$$

where γ is an abstract constant. For $\nu=1$ we have plane waves; for $\nu=2$, cylindrical waves; and for $\nu=3$, spherical waves.

If $\gamma = c_p/c_v$, the equations (3) and (4) may be regarded as describing the adiabatic motions of a gas where the entropy can be different for different particles.

Substituting the formulae (1) in (3) and (4) we get:

$$\left.\begin{aligned} &\lambda\left[(n+mV)V' + m\frac{P'}{R}\right] = V^2 - V - (k+1)\frac{P}{R} \\ &\lambda\left[mV' + (n+mV)\frac{R'}{R}\right] = -s - (k-\nu+3)V \\ &\lambda(n+mV)\left[\frac{P'}{P} - \gamma\frac{R'}{R}\right] = -s(1-\gamma) - 2 - [k(1-\gamma) + 1 - 3\gamma]V \end{aligned}\right\} \qquad (5)$$

* It is apparent that for $n \neq 0$ only the ratio m/n is relevant.

** v_0, ρ_0, p_0 can be considered to be functions of r, taking constant values for $r \neq 0$ and $r \neq \infty$; and in the general case having points of discontinuity for $r=0$ and $r=\infty$.

If the constant b is absent, the left numbers of equations (5) are to be replaced by zeros, with the result that we shall have three simple finite relations connecting V, P, R, k, s. So in this case formulae (1) give a family of exact solutions of equations (3) and (4) which depend upon two arbitrary constants. Substituting t with $t-t_0$ we shall obtain a solution that depends upon three arbitrary constants.

For $\nu=1$ the coordinate r in formulae (1) can be replaced by $r-r_0$, whereupon we obtain a solution depending on four arbitrary constants.

The integration of the system of equations (5) can be reduced to the integration of a single equation of the form

$$d\left(\frac{P}{R}\right)/dV=\frac{P}{R}f\left(V, \frac{P}{R}\right) \tag{6}$$

where $f\left(V, \frac{P}{R}\right)$ is a fractionally linear function of P/R and a simple rational function of V.

Other more complicated mathematical operations are involved in the method suggested by Bechert [1] for obtaining exact solutions of equations (3) of the same type in the particular case where equation (4) is replaced by the finite relation

$$p=c\rho^{\gamma}$$

where c is a constant.

The exact solutions we have obtained can be used for solving a number of important problems, some of which will be discussed elsewhere.

M. V. Lomonosov State University.
Moscow.

Received
14. VI. 1944.

REFERENCES

[1] K. Bechert, Ann. d. Phys., 39, H. 3 (1941).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

MATHEMATICAL PHYSICS

ARTIFICIAL MANY-DIMENSIONAL MODELS FOR APPROXIMATE SOLUTIONS OF INTEGRAL EQUATIONS

By L. I. GUTENMACHER

(Communicated by N. G. Brujevitch, Member of the Academy, 16.XII. 1944)

The methods of electrical space and time modelling of physical phenomena developed by the author ([1-4]) were based on the mathematical apparatus of difference equations (the method of Runge). It was pointed out, however, that the time derivatives in the equations are not replaced by divided differences.

Further investigations of processes in our many-dimensional artificial models showed that the apparatus of integral equations is better adapted to the description of the phenomena in models. At the same time we find many important properties of the models essential for the theory and practice of modelling.

For the construction of models we use bipoles (Fig. 1), which may consist of arbitrary combinations of concentrated elements of electrical lines — resistances R, inductances L and capacities C. The transitional conductance A or B of such a bipolar scheme (Fig. 1) is given by the expression

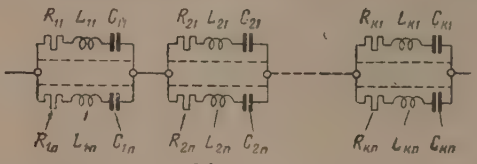


Fig. 1

$$A,\ B = 1 : \sum_{i=1}^{n} \left[1 : \sum_{m=1}^{k_i} \frac{1}{R_{im} + L_{im}\, p + \frac{1}{C_{im}\, p}} \right]$$

where p is the operator $\frac{\partial}{\partial t}$.

In Fig. 2 is represented a part of a four-dimensional electrical scheme situated in a plane (cf. ([1-4])). We describe the properties of the model by means of the function of the connexion resistance Z. To this end we leave in the model only the connexion conductivities A and choose among the junction-points of the model or outside of the model a point of zero potential O.

Apply at one of the junctions with the coordinates $\{s_{lj_l}\} = (s_{1j_1}, s_{2j_2}, \ldots, s_{nj_n})$ a source, *i. e.* a source of current $I\{s_{lj_l}\}$. Through the elements A of the model passes a current to the point O, and in the junctions of the model $\{x_{ki_k}\} = (x_{1i_1}, x_{2i_2}, \ldots, x_{ki_k})$ arise potential differences $u\{x_{ki_k}\}$. The ratio of

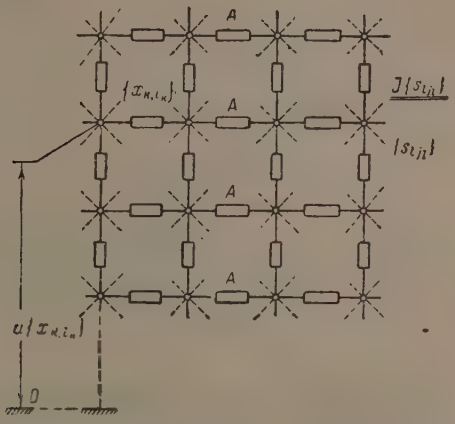


Fig. 2

these potential differences to the current strength $I\{s_{lj_l}\}$ has the dimensionality of resistance

$$\frac{u\{x_{ki_k}\}}{I\{s_{lj_l}\}} = Z\{x_{ki_k};\ s_{lj_l}\} \qquad (2)$$

whence we obtain a law similar to Ohm's law:

$$u\{x_{ki_k}\} = Z\{x_{ki_k};\ s_{lj_l}\}\ I\{s_{lj_l}\}\ (k,\ l = 1,\ 2,\ \ldots,\ n)$$

The function $Z\{x_{ki_k};\ s_{lj_l}\}$ takes into account also the outer (boundary resistances connecting boundary junctions of the model together with the chosen zero point O.

Under the simultaneous action of positive and negative sources in all or a part of junctions of the scheme the potential differences at all points of the scheme may be determined according to the principle of superposition valid for linear systems:

$$u\{x_{ki_k}\} = \sum_{j_n=1}^{m_n} \cdots \sum_{j_1=1}^{m_1} Z\{x_{ki_k};\ s_{lj_l}\}\, I_1^1\{s_{lj_l}\} \qquad (k,\ l = 1,\ 2,\ \ldots,\ n) \qquad (3)$$

The Nature of Sources. The currents I flowing to or from the junctions of the scheme to the zero point of the system can exist only in the presence of special conductances B connecting the junction directly with the zero point. In the general case each such conductance may consist of R, L and C connected according to the scheme in Fig. 1, and in certain branches may be introduced a source of emf E.

Let to every junction be connected a conductance B without sources, and in parallel to it a rather small conductance B_0 with a source E (4) (cf. in Fig. 3 the scheme of connexions for one of the arbitrary junction). Choose the values of conductances proportional to the «volumes» of elementary parallelepipedons

$$\overline{B} = B\Delta s_1 \Delta s_2 \ \ldots\ \Delta s_n, \quad \overline{B}_0 = B_0 \Delta s_1 \Delta s_2 \ \ldots\ \Delta s_n \qquad (4)$$

The conductances B determine the dependence between the potential differences at the functions $u\{s_{lj_l}\}$ and the currents $I\{s_{lj_l}\}$. By Ohm's law

$$I\{s_{lj_l}\} = \bar{B}\{s_{lj_l}\}\, u\{s_{lj_l}\} + \bar{B}_0\{s_{lj_l}\}\,[E\{s_{lj_l}\} - u\{s_{lj_l}\}] \tag{5}$$

Substituting the values of the intensities of sources into the equation (3) we obtain

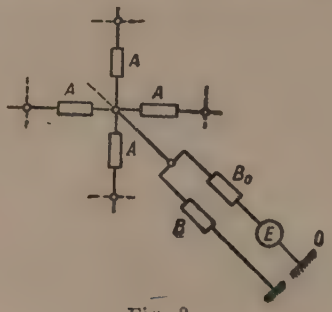


Fig. 3

$$u\{x_{ki_k}\} = F\{x_{ki_k}\} + \sum_{j_n=1}^{m_n} \cdots \sum_{j_1=1}^{m_1} Z\{x_{ki_k};\ s_{lj_l}\}\,[B - B_0]\{s_{lj_l}\}\ u\{s_{lj_l}\}\,\Delta s_1\,\Delta s_2 \ \ldots\ \Delta s_n \tag{6}$$

where

$$F\{x_{ki_k}\} = \sum_{j_n=1}^{m_n} \cdots \sum_{j_1=1}^{m_1} Z\{x_{ki_k};\ s_{lj_l}\}\,B_0\{s_{lj_l}\}\,E\{s_{lj_l}\}\,\Delta s_1\,\Delta s_2 \ \ldots\ \Delta s_n.$$

When the number of the elements A and B of the scheme tends to infinity, *i. e.* in the limit $\Delta s_k \to 0$, we obtain a linear integral equation of the second kind

$$u\{x_k\} = F\{x_k\} + \int_{(G)} \cdots \int Z\{x_k;\ s_k\}\,\underline{B}\{s_k\}\,u\{s_k\}\,ds_1\,ds_2 \ \ldots\ ds_n \tag{7}$$

Herein $\underline{B} = B - B_0$, and the integration extends over the domain G, the points of which are determined by the coordinates s_k $(k = 1, 2, \ldots, n)$.

We use here the main idea of Fredholm's method, which consists in that the integral equation of the type (7) is considered as the limit of a system of linear equations (6) when the number of junctions increases to infinity.

In the following note we propose to consider a number of important operation schemes of electrical models as depending on the structure of the parameters of bipolars A and B, and the properties of the sources E.

Received
16. XII. 1944.

REFERENCES

[1] L. I. Gutenmacher, C. R. Acad. Sci. URSS, XXVII, No. 3 (1940). [2] Л. И. Гутенмахер, Электричество, № 5 (1940). [3] Л. И. Гутенмахер, ЖТФ, XII, 2—3, 47 (1942). [4] Л. И. Гутенмахер, Электрическое моделирование изд. АН СССР, 1943.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

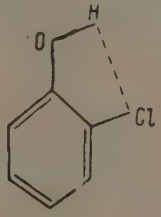
PHYSICS

A STUDY OF THE INTRAMOLECULAR HYDROGEN BOND OF GUAIACOL BY THE METHOD OF THE COMBINATION SCATTERING OF LIGHT. III

By M. I. BATUEV

(Communicated by A. N. Nesmeyanov, Member of the Academy, 11. VII. 1944)

The study of the combination scattering of light spectra of *o*-chlorophenol [1] led us to the conclusion that beside the benzene ring this molecule has another five-membered intramolecular one, the formation whereof depends on the interaction between H and Cl, which, undoubtedly, is not of the nature of a valent link, but is similar to intramolecular interaction of the hydrogen bond type:



However, it should be pointed out that no strong intermolecular interaction is usually observed between the hydroxyl group and the solvent which comprises

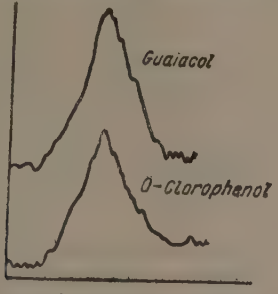


Fig. 1. Microphotometer tracings of the scattering bands of OH group of guaiacol and *o*-chlorophenol

chlorine in its molecule [2]. It is, therefore, interesting to study the scattering spectra of such disubstituted phenols which possess in ortho-position an atom with a strong tendency to form intermolecular hydrogen bond with a hydroxyl group.

Oxygen is such an atom and, for that reason, guaiacol was selected as a suitable object for further investigation. In the present study the scattering spectra of guaiacol and of its benzene or acetone solutions have been investigated.

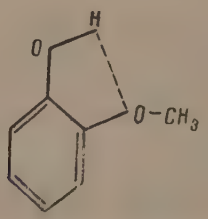
The scattering spectra obtained by the author have shown that the normal vibrations of the hydroxyl group of guaiacol closely resemble the respective oscillations of *o*-chlorophenol. In both cases we have obtained a narrow band, the maxima of intensity for the two compounds corresponding to the same frequency. This maximum is displaced by more than 100 cm^{-1} compared with that of the bands which characterize the identical hydroxyl group of phenol or *p*-chlorophenol; at the same time the latter bands appear to be much broader (Fig. 1 and Table 1).

Table 1

Position of the Middle of the Band of the Hydroxyl Group in Phenol, *p*- and *o*-Chlorophenol and Guaiacol, in cm^{-1}

Compounds	Frequencies	Width of the band*
Phenol	3410±15	250
p-chlorophenol . . .	3420±15	300
o-chlorophenol . . .	3535±15	120
Guaiacol	3535±15	110

Thus, it must be admitted that the hydroxyl group in the molecule of *o*-chlorophenol and the hydroxyl group of guaiacol are under very similar conditions, and so the formation of intramolecular five-membered ring due to hydrogen bond formed between OH and H should be supposed to take place:



This conclusion is corroborated by our studies of the spectra of the solutions of guaiacol and *o*-chlorophenol in benzene or carbon tetrachloride.

As is generally recognized ([1], [2]), such solutions at sufficiently low concentrations furnish a means to obtain data upon isolated molecules of the dissolved substance, since they are protected from interaction with each other by dilution, and since no perceptible influence of molecules of such a solvent as benzene upon hydroxyl group is observed. It has been found that for 12 per cent benzene solution of guaiacol, as well as for the corresponding solution of *o*-chlorophenol in carbon tetrachloride these spectra are characterized by an exceedingly narrow band, resembling a distended line. The frequency of this band-line for both *o*-chlorophenol and guaiacol practically coincides with the frequency of the maximum of the band of the hydroxyl group of liquid guaiacol, and is displaced 60 cm^{-1} toward a lower frequency compared to the frequency of vibrations of the hydroxyl group in isolated molecules of phenol and *p*-chlorophenol (Table 2).

* A rough measurement over the microphotometer tracings of the band obtained from the exciting triplet: 27388, 27353, 27293 cm^{-1}.

Table 2

Combination Frequencies of OH Group in the Solutions of Phenol, *p*- and *o*-Chlorophenol in CCl_4 and of Guaiacol in Benzene*, in cm^{-1}

Compounds	Frequencies
12 to 15 p. c. solution of phenol in CCl_4	3603±5
12 to 15 p. c. solution of *p*-chlorophenol in CCl_4 .	3602±5
12 to 15 p. c. solution of *o*-chlorophenol in CCl_4 .	3540±5
12 to 15 p. c. solution of guaiacol in benzene . . .	3540±5

In this way we come to a conclusion that in guaiacol molecule, just as in that of *o*-chlorophenol, an internal five-membered ring is formed, thus sufficiently protecting the hydroxyl group from any intermolecular interaction with the surroun-

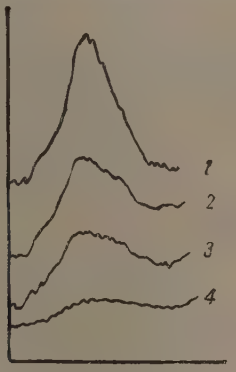


Fig. 2. Microphotometer tracings of the scattering bands of hydroxyl group in acetone solutions of guaiacol: *1*, guaiacol; *2*, guaiacol +15 p. c. of acetone; *3*, guaiacol+40 p. c. of acetone; *4*, guaiacol +50 p. c. of acetone

ding molecules, which usually results in the diffusion of the hydroxyl line to a broad band. The frequency of the hydroxyl group incorporated in the five-membered ring is characteristic of the latter (3540 cm^{-1}) and remains unaltered on transition to isolated molecules of guaiacol and *o*-chlorophenol. The slightly greater width of the band for pure guaiacol and *o*-chlorophenol in comparison with that for their solutions seems to indicate that some perturbation of OH vibrations by the surrounding molecules takes place, which like all intermolecular interaction causes the band to diffuse owing to unavoidable fluctuations in the extent of that interaction. However, the presence of the ring provides a sufficient protection for the hydroxyl group and the perturbing interaction is relatively slight.

Essentially different results are obtained when guaiacol (as well as *o*-chlorophenol) is dissolved in acetone. In this case, with increasing concentrations of acetone the narrow band of the hydroxyl group of guaiacol diffuses gradually to a broad band, displaced toward lower frequency and characteristic of solutions of phenol or alcohol in acetone. At a 25 per cent concentration of acetone the peak of the band of the hydroxyl group of liquid guaiacol (3535 cm^{-1}) is still distinct, but at a 40 p. c. concentration it disappears almost altogether, and at a 50 p. c. con-

* The exciting triplet: 27388, 27353, 27293 cm^{-1}.

centration of acetone there is no trace of it left—only a wide band diffused toward lower frequencies presents itself to the eye. By analogy with the corresponding *o*-chlorophenol experiments it may be supposed that the diffusion of the narrow band to a wide one in the direction of lower frequencies is due to the cleavage of the five-membered intramolecular ring of guaiacol in a greater and greater number of molecules, and to the substitution of the intramolecular hydrogen bond O—H... O by the intermolecular hydrogen bond O—H...O.

Accordingly, all of the evidence furnished by the present investigation of the spectrum of guaiacol, confirms the assumption that a five-membered ring is formed in guaiacol as well as in *o*-chlorophenol. However, such a great resemblance in the data obtained for the two molecules is somewhat unexpected. The data on intermolecular interaction suggested a different rôle of chlorine and oxygen in ortho-position and caused to anticipate rather more efficient influence upon hydroxyl group in the case of guaiacol molecule than in that of *o*-chlorophenol. It should be remembered that, as shown by the spectrum of *o*-cresol, investigated by the author ([1]), the presence in ortho-position of such a neutral group as methyl has no effect whatever upon OH group which behaves in the same way as it does in *p*-cresol or phenol.

It may be hoped that more will be learned about the nature of the interactions under discussion by varying the atoms in the investigated five-membered rings having perturbing effect upon the hydroxyl group, by altering the geometry of the ring (by passing to six-membered from five-membered rings) and by investigating the spectra of *o*-chlorophenol, guaiacol and other corresponding substances and their solutions in various solvents and at various temperatures.

The author's sincerest gratitude is due to Prof. G. S. Landsberg for his discussion of the obtained data.

Institute of Organic Chemistry.
Academy of Sciences of the USSR.

Received
11. VII. 1944.

REFERENCES

[1] M. I. Batuev, C. R. Acad. Sci. URSS, XL, No. 7 (1941). [2] V. I. Malyshev, Bull. Acad. Sci. URSS, sér. phys, 5, 13 (1941).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

PHYSICS

ON CURRENTS OF FAST CHARGED PARTICLES INDUCED BY REVOLVING MAGNETIZED COSMIC BODIES

By J. P. TERLEZKY

(Communicated by S. I. Wawilow, Member of the Academy, 5.II. 1945)

Owing to different orientations of the magnetic moment $\vec{\mu}$ and the axis of revolution of the Earth or the Sun, in the space surrounding these cosmic bodies, besides the magnetic field

$$\vec{H} = \frac{3\vec{r}(\vec{\mu}\vec{r}) - \vec{\mu}r^2}{r^5} \tag{1}$$

an electric field

$$\vec{E} = \frac{1}{cr^3}[\vec{r}[\vec{\omega}\vec{\mu}]] \tag{2}$$

is also induced; here $\vec{r}$ is the radius-vector connecting the centre of the body with the point of observation; $\vec{\omega}$ is the cycle frequency vector of revolution; c is the velocity of light.

The electric field (2) should accelerate the charged particles located in its neighbourhood, and this may possibly lead to appreciable cosmic phenomena.

According to (1) and (2) near the surface of the bodies $E \ll H$, and therefore charges emerging from the surface with small initial velocities can practically be considered as travelling along tubular surfaces of small radius enclosing the magnetic lines of force*. On the average, if we disregard the revolution of the charge on the surface of the tube about the axis of the latter, one may consider that the charge is accelerated along the corresponding magnetic line of force.

The energy $\mathcal{E}$ accumulated by the charge in the movement is determined by the work the charge does in the electric field (2) while on its way along the magnetic line of force.

Carrying out the computations for a charge emerging from the surface of the body and reaching a point with the coordinates r, ϑ, φ, we obtain the expression

$$\mathcal{E} = \frac{e}{c}\omega\mu\sin\Theta\left|\frac{\cos\varphi\cos\vartheta}{r}\left[\sin\vartheta \mp \sqrt{\frac{r}{r_0} - \cos^2\vartheta}\right]\right| \tag{3}$$

where r is the distance from the centre of the body; ϑ is the magnetic latitude angle referred to the magnetic equator; φ is the magnetic longitude

* The basis for this picture of motion of the charges is published by the author elsewhere.

referred to the meridian passing through the magnetic and geographical poles; r_0 is the radius of the cosmic body; e, particle charge; Θ, angle between directions of $\vec{\mu}$ and $\vec{\omega}$. In the expression just obtained the minus sign corresponds to the case where the trajectory of the charge does not intersect the plane of the magnetic equator; otherwise the plus sign should be taken.

According to (3), the energy of the charges returning to the surface of the body is given in electron volts by the expression

$$V = 3.6 \cdot 10^{-3} \frac{H_m r_0^3 \sin\Theta}{T} \left| \sin 2\vartheta \cos\varphi \right| \qquad (4)$$

where H_m is the maximum magnetic field at the surface of the body expressed in gausses; r_0 is the radius of the body in kilometres; T is the period of revolution in days. From (4) one gets that the maximum energy attainable in the field of the Earth ($H_m = 0.7$ gauss, $r_0 = 6.4 \cdot 10^3$ km, $T = 1$ day, $\Theta = 18°$) is $3 \cdot 10^4$ eV; in the field of the Sun ($H_m \approx 50$ gauss, $r_0 = 7 \cdot 10^5$ km; $T = 31$ days; $\Theta = 6°$) the value is $2 \cdot 10^8$ eV.

The effect suggested here is possibly related to a number of helio- and geophysical phenomena. Moreover, there is no reason to suppose that magnetization and different orientations of $\vec{\mu}$ and $\vec{\omega}$ are inherent properties of the Sun and Earth alone among cosmic bodies. If one assumes that on other stars the magnetic field is of the same order and nature as that of the Sun, then it may be shown that among some of the larger familiar stars there are quite a large number which may induce currents of particles with energies of the order of that of the cosmic ray particles.

M. V. Lomonosov State University.
Moscow.

Received
5. II. 1945.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

CHEMISTRY

SOLUBILITY OF POLYMERIZING HYDROCARBONS (MONOMERS) IN AQUEOUS SOLUTIONS OF EMULSIFYING AGENTS

By A. I. YURZHENKO and S. M. MINTZ

(Communicated by A. E. Arbuzov, Member of the Academy, 2.VIII. 1944)

As is well known, the reciprocal solubility of the hydrocarbon and the aqueous phases increases considerably when surface-active agents (of an intermediate polarity) are introduced into the system. Smith (1) has observed that in a 0.4 *N* solution of sodium oleate the solubility of many organic liquids increased by about 10 per cent. An increase in the solubility of hydrocarbons in water under the action of surface-active substances has been investigated by Rehbinder and co-workers (2), as well as by Lawrence (3). These authors have worked out concepts of spontaneous emulsifying and colloidal solubility and interpreted the mechanism of the formation of so-called «soluble oils». The works of Semenchenko, Davidovskaya and Ivanova (4) deal with such cases where surface-active substances are molecularly soluble in water and do not lead to the formation of colloidal systems.

When hydrocarbons (or rubber-forming substances in general) are being polymerized in emulsions, the process by which the monomer under the action of an emulsifying agent passes from droplets of the emulsion into the aqueous phase does certainly take place, such a solubility being perhaps of a colloidal character. In the study of the influence that emulsifying agents have upon the process of polymerization in emulsions (the formation of synthetic latex) it is very important to characterize this hydrotropic action of the emulsifying agents used, and this has been the aim of the present investigation. The results obtained made it possible to develop new views on polymerization in emulsions and its topography.

A very successful method of investigation proved to be the determination of the refractive index of the aqueous phase of the emulsion in a thin layer, using the Abbé refractometer. This led to the possibility of carrying out measurements in the presence of droplets of the emulsified hydrocarbon, which made the system opaque in a thick layer. These droplets, having a much higher refractive index compared to that of the aqueous phase, scattered the falling rays and could not influence the readings of the refractometer. The prisms of the latter were wetted in this case only by the exterior (aqueous) phase.

The mixture of the hydrocarbon and the soap were shaken mechanically for 30 min. at 20° C. Special experiments had shown this to be quite sufficient for reaching equilibrium. The monomer introduced into the vessel was used up in part to saturate the gaseous space in the vessel and to saturate the aqueous phase; its excess was emulsified. With an increase of the amount of the monomer taken complete saturation of the aqueous phase took place and the refractive index reached its limit (Fig. 1). It is easy to see that further addition of the hydrocarbon, leading to the formation of more and more concentrated emulsions, has no influence on the changes of n_{max}, this certifying the reliability of the method.

The concentration of the hydrocarbon dissolved in the aqueous phase v_s (in per cent by volume) was calculated by using the formula:

$$v_s = \frac{\rho - \rho_0}{\rho_1 - \rho_0} 100$$

ρ, ρ_0 and ρ_1 being the expression $(n^2-1)\ (n^2+2)$ for the aqueous phase of the emulsion (n_{max}), the pure solution of the emulsifying agent and the pure monomer, correspondingly.

The data obtained make it possible to dispose the alkaline emulsifying agents investigated at different concentrations and at different pH of the medium into

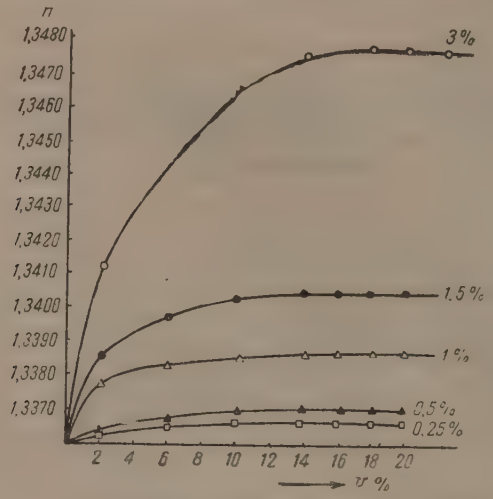


Fig. 1. Saturation of sodium oleate by isoprene ($c_{Na_2CO_3}$=0.16 equiv./l)

the following sequence, according to their capacity of increasing the solubility of hydrocarbons in the aqueous phase: potassium (sodium) oleate > ammonium

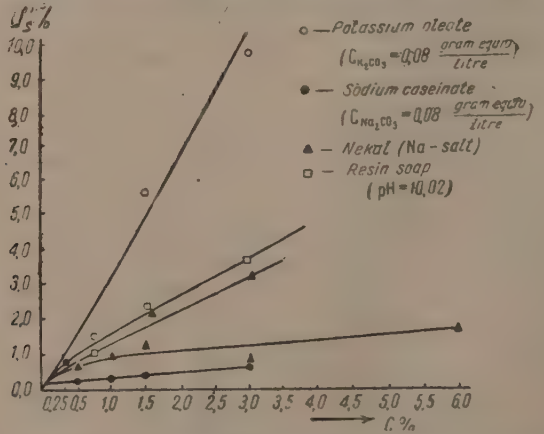


Fig. 2. Solubility isotherms of isoprene in relation to the emulsifying agent

oleate > resin soap (Na salt) > nekal (sodium butyl α-naphthalene-sulphonate) > >sodium caseinate (Fig. 2).

It should be noted that sedimentation analysis of the emulsions obtained showed that the emulsifying activity of the agents mentioned (with the excep-

tion of the caseinate) lays within much narrower limits compared to their hydrotropic action. Such an emulsifying agents as nekal sometimes gave finer emulsions than sodium oleate, although its hydrotropic action is incomparably lower compared with the oleates.

As shown in Fig. 2 for isoprene and in Table 1 for other monomers, their hydrotropic activity increases almost as a linear function of concentration. It is essential to note that those concentrations of the emulsifying agents which are used when polymerization is effected in an emulsion (3—5 per cent) bring into the water phase a considerable amount of the monomer (up to 10—15 per cent).

Table 1

Solubility of Different Rubber-forming Substances in Solutions of Potassium Oleate ($cK_2CO_3 = 0.32$ g. equiv./l)

Soap concentration, %	v_s		
	styrene	nitrile	isopreue
0.75	0.62	16.2	0.65
1.50	2.2	16.7	3.5
3.00	6.8	17.3	9.0

The mechanism of the solution of hydrocarbons in dilute solutions of soaps is, no doubt, to a considerable degree of a colloidal nature. According to the modern notions on the solutions of soaps ([5]), this process should be regarded not as the molecular distribution of the hydrocarbon passing over into the water phase, but as its concentration chiefly in the micelles of the soaps (in the zone where nonpolar parts of their molecules accumulate). This leads to the conclusion that such a solubility should depend on the colloidal state of the soap.

Actually the change in the concentration of the electrolyte(free base) changes sharply the hydrotropic action of the soap. This can be seen, for instance, from data obtained for sodium oleate and isoprene (Table 2). The pH of the medium in this case was changed by changing the content of free sodium carbonate.

Table 2

Influence of Free Sodium Carbonate on the Solubility of Isoprene in Aqueous Solutions of Sodium Oleate

Concentration of free sodium carbonate, g. equiv./l	v_s				
	Soap concentration, %				
	0.25	0.50	1.0	1.5	3.0
0.08	0.45	0.63	0.9	3.22	10.1
0.16	0.68	0.75	1.7	3.0	8.4
0.32	0.50	0.75	1.33	1.55	5.5

The solubility of the hydrocarbon in the soap solution decreases under the influence of the free base the more rapidly, the higher is the concentration of the soap.

Laboratory of Physical Chemistry.
S. V. Lebedev Experimental Plant No. 743.

Received
2. VIII. 1944.

REFERENCES

[1] L. Smith, J. phys. Chem., **36**, 1401, 1672, 2455 (1937). [2] P. Rehbinder, Bull. Acad. Sci. URSS, sér. chim., **5**, 639 (1936); K. Pospelova and P. Rehbinder, Acta Physicoch. URSS, **16**, 71 (1942); V. Mokiyevski and P. Rehbinder, ibid., **13**, 97 (1940). [3] A. S. C. Lawrence, Trans. Far. Soc., **33**, 815 (1937). [4] В. В. Семенченко и Т. Н. Иванова, ЖОХ, **5**, 807 (1935); В. В. Семенченко и К. П. Давидовская, ЖОХ, **4**, 632 (1934). [5] G. S. Hartley, Aqueous Solutions of Paraffin Chain Salts, Paris, 1936.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

PHYSICAL CHEMISTRY

INFLUENCE OF THE VOLUME SORPTION OF VAPOURS ON THE SOLIDIFICATION TEMPERATURE OF POLYMERS

By S. N. ZHURKOV and R. I. LERMAN

(Communicated by A. F. Joffe, Member of the Academy, 29. VII. 1944)

The temperature range of the variation of the properties of polymers upon transition from a rubber-like state to a solid glass-like sharply depends on the non-polymerized low molecular fractions dissolved in them and on artificially introduced foreign substances—plastificators. When dissolved in the polymer, they form together an overcoolable system, lower the solidification temperature and change the mechanical characteristics. This opens the possibility for controlling the behaviour of polymers, conformably to the conditions of their use in industry.

In spite of the importance of this problem both from the scientific and practical point of view, we have for the present no generally accepted theory of the action of plastificators.

1. Experimental part. Our experiments were carried out with volatile organic solvents of a relatively simple structure, which were introduced into the polymer when the latter was swollen in the vapours of these substances. By thus simplifying the conditions of plastification it was possible to investigate the fundamental regularities in the action of plastificators.

The influence of solvents has been studied on four polymeric substances. Two of them, the copolymer of butadiene with nitryl of acrylate acid and polymethyl-metacrylate contain in the polymeric chain dipole groups $\left(-CN;\ -C\begin{matrix}O\\ \diagdown\\ OCH_3\end{matrix}\right)$. The two other polymers—butadiene rubber and polyisobutylene are non-polar.

The solidification range was studied directly by measuring the hardness of the polymer at various temperatures according to the depth of the impression left by a metallic sphere (1).

In order to exclude the influence of the time factor, the measurements were carried out in an identical time range. The values of the high-elastic modulus calculated with the help of the Hertz's formula, which are given below, correspond to a deformation of the specimens during 30 seconds. Fig. 1 gives the logarithm of the modulus as a function of the temperature for the above-mentioned four polymers.

Upon swelling the temperature region of a sharp variation of the elastic modulus is displaced towards lower temperatures. The experiments have shown that the influence of volatile solvents is reduced to shifting the S-shaped curves for the pure polymer parallel to the temperature axis without any change of shape; the magnitude of the temperature shift increases with the concentration of the sorbed solvent.

As the curve $\lg E = f(t)$ shifts parallel to itself, the magnitude of the shift ΔT (Fig. 2) may be used as a quantitative characteristics of the influence of the solvent on the solidification temperature of the polymer.

For all four polymeric substances, with over 30 various solvents, the solidification temperature is found experimentally to depend essentially on the number of sorbed molecules.

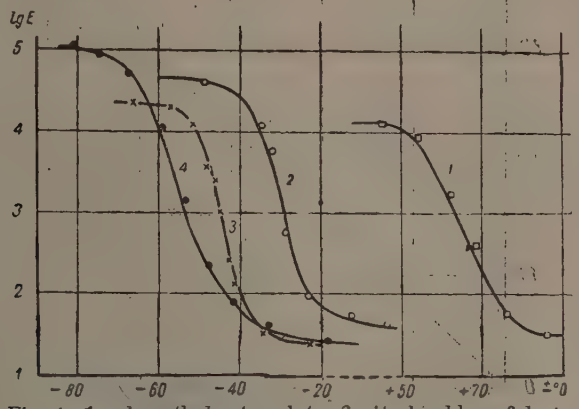


Fig. 1. *1*, polymethylmetacrylate; *2*, nitryl rubber; *3*, butadiene rubber; *4*, polyisobutylene

The simplest regularity is observed in polar polymers swelled in the vapour of dipole or strongly polarizable liquids. In this case the lowering of the temperature proves to be independent of the molecular volume, structure, size of the absorbed molecule or its chemical nature, and is determined solely by the number of sorbed particles. For comparatively small concentrations of the dipole solvent absorbed by

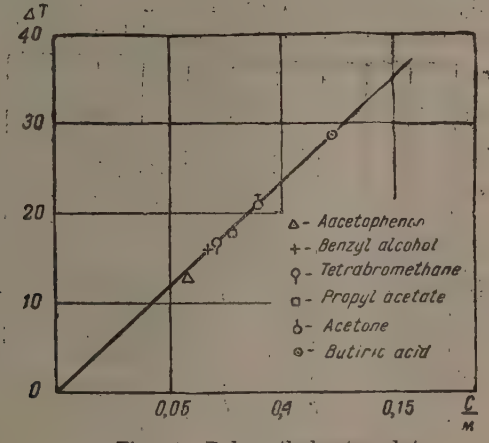


Fig. 2. Polymethylmetacrylate

the polymer the decrease in temperature ΔT varies directly as the number of absorbed molecules and is satisfactorily expressed by the equation

$$\Delta T = kn$$

where n denotes the number of sorbed molecules, and k is a constant coefficient independent of their nature.

This reminds one of Raoul's equation for dilute solutions, although no phase transformations have been observed thus far upon the transition of non-crystallizing polymers from a rubber-like state into a solid one (²).

The experimental data obtained are represented in Figs. 2 and 3. What is plotted on the abscissa axis is a quantity proportional to the number of sorbed

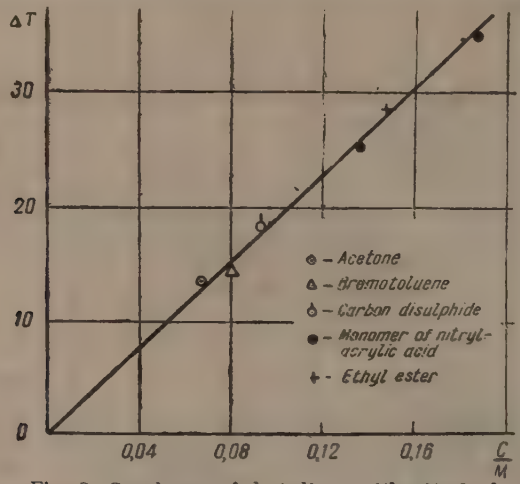


Fig. 3. Copolymer of butadiene with nitryl of acryl acid (33 per cent)

particles, namely the ratio of the weight concentration (in per cent) to the molecular weight of the solvent (c/M).

The coefficient k, calculated from the slope of the straight lines, turned out to be 230 for polymethylmetacrylate, 190 for nitryl rubber (in degrees per mole of the sorbent solvent in 100 g of the polymer).

For non-polar polymers, butadiene rubber and polyisobutylene the relation was found to be more complicated. The decrease of the solidification temperature

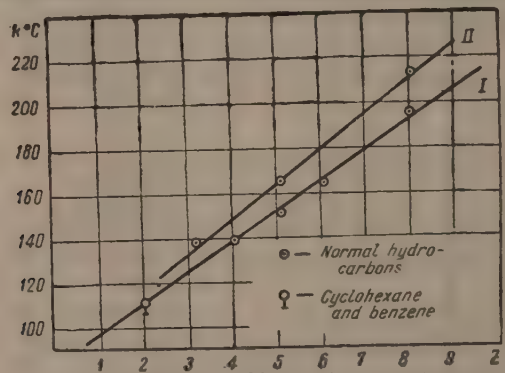


Fig. 4. *I*, butadiene rubber; *II*, polyisobutylene

is related in this case not only to the number of sorbed molecules, but also to their magnitude and configuration. The value of the coefficient k does not remain constant, but rises in the sorption of butane, pentane, hexane and octane with the increase of their molecule and of the number of carbon links. Fig. 4, where the coefficient k is plotted as a function of the number of carbon links, shows that for normal saturated hydrocarbons k is proportional to Z.

The plastificating action of a given molecule depends, however, not only on the number of links in the sorbed molecule, but also on their mutual orientation.

In the case of an equal number of carbon atoms the linear molecule (n-hexane) produces a markedly stronger action than the cyclic one (cyclohexane and benzene). The latter are situated in Fig. 3 at $Z=2$, although the number of carbon links amounts to 6.

2. Discussion of the Experimental Results. The notable influence of the number of sorbed molecules on the solidification temperature, the unresponsiveness of this effect to the chemical nature of the solvent and the different behaviour of polar and non-polar polymers, is, in our opinion, the result of the blocking by the solvent of separate portions of the polymeric molecule.

The monomolecular and selective character of the sorption process leads to the particles that diffuse into the polymer entering into interaction with definite groups of the polymer and forming with them sorption molecular compounds ([3–7]). Such sorption complexes sharply disturb the interaction between the chains. Separate portions of the polymeric molecule, carrying sorbed particles, are blocked by the latter and, as a result, their bond with the neighbouring groups is weakened.

From this point of view the accurate validity of the relation $\Delta T = kn$ for polar polymers is explained by the fact that each molecule of the solvent blocks a single dipole group of the polymer. The screening of the electrical field of a dipole group turns out to be so complete here, as to be equivalent to the exclusion of this group from dipole interaction with other polar groups of the neighbouring polymer molecules.

The independence of this effect of the structure and chemical nature of the molecules of the polar solvent is in this case obvious, since only polar portions of the sorbed molecules interact with each other. The case of non-polar polymer is different. An arbitrary carbon link of their chain molecules can be considered as an active point which can adsorb the solvent.

For the same reason in the molecule of a non-polar solvent several active points where it can be held simultaneously should be supposed to be present. Under such circumstances it is natural to expect that the probability of a simultaneous blocking of several groups of the polymer will increase with the size of the sorbed molecule (Fig. 4).

The above assumption as to the action of plastificators is in good agreement with the experimental data obtained by us and enables one to understand the molecular mechanism of solidification of polymeric substances.

Physical-Technical Institute.
Academy of Sciences of the USSR.
Leningrad.

Received
29. VII. 1944.

REFERENCES

[1] L. Larrick, Phys. Rev., **57**, 358 (1940). [2] R. Pummerer u. G. Susich, Kautschuk, **7**, 117 (1931). [3] F. Goucher and H. Ward, Phil. Mag., **44**, 1002 (1922). [4] C. Trogus, Tomonarie u. K. Hess, Z. physik. Chem. (B), **16**, 55 (1932). [5] V. Kargin u. S. Papkov, Acta Physicoch. URSS, **3**, 839 (1935); **7**, 667 (1937). [6] А. Тагер и В. А. Каргин, ЖФХ, **15**, 1036 (1941). [7] W. Baker, C. Fuller and N. Pape, J. Am. Chem. Soc., **64**, 776 (1942).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

PYSICAL CHEMISTRY

FORMATION OF MONOMICELLAR FILMS OF COLLOIDAL HYDROXIDES OF METALS ON THE SURFACE OF HYDROSOLS

By S. G. MOKRUSHIN

(Communicated by A. N. Frumkin, Member of the Academy, 7.XII.1944)

It has been shown by the author that the formation of monomicellar layers (films) on the surface of colloidal solutions takes place spontaneously when the latter are undistributed ([1]). This phenomenon is associated chiefly with positively charged colloids which coagulate on the surface of the solution, and as a result the latter gradually becomes covered with a monomicellar condensed (solid) film. The thickness of the film that has formed as a result of coagulation is of the order of a micella, *i. e.* the film is monomicellar, similar to the monomolecular films of really dissolved surface-active substances.

On the basis of an experimental investigation of the thicknesses of films obtained by the foregoing method, the author established that they actually have a limiting value, namely, that of a monomicellar film. To determine the limiting thickness, a study was made of the kinetics of formation of the surface films from colloidal hydroxides of metals—iron, aluminium and chromium. The most thoroughly investigated was the film-formation kinetics for the colloidal hydroxide of iron on a corresponding hydrosol. The iron hydroxide hydrosol was obtained by the method described by Krecke. The thickness of the film of iron hydroxide was determined by the Langmuir-Blodgett ([3]) method, modified by the author to make it applicable to the present investigation. Essentially this method consists in building up a polymolecular film on a polished chromium plated plate by successively raising and lowering the latter above and below the surface of the solution until it exhibits interference colours, and then getting its thickness from the minimum reflection of monochromatic light. By dividing this quantity by the number of monomolecular films, it is possible to compute the thickness of the latter.

The following data for calcium stearate (Table 1) were taken from Langmuir and Blodgett's investigations of the thicknesses of polylayer films having a definite colour.

Employing the above method, the author obtained polylayer films of colloidal iron hydroxide by successive submersion of a polished chromium plate in a colloidal solution of the hydroxide. Upon submersion, the upper (outer) surface of the surface film attached itself to the dry surface of the plate, since it has hydrophobic properties ([4]). Castor oil served as «piston oil».

When the slide was withdrawn, the surface film again attached itself to it, but this time by its lower, *i. e.* hydrophilic surface, adhering very feebly and unevenly.

Then the slide with the two films —the firmly attached one (attached during submersion) and the feebly attached one (attached during withdrawal)—was washed with a jet of distilled water. The feebly attached upper film was easily

Table 1

Colour and Thickness of Polylayer Films of Calcium Stearate

Colour of films	Number of mono-layers	Thickness of film (in Å)
Yellowish-brown . .	21	512.4
Dark-blue	41	1000.4
Light-blue	61	1488.4
Yellow	81	1976.4
Red	101	2464.4
Blue	121	2952.4
Green	141	3440.4
Reddish-yellow . . .	161	3928.4

washed off, whereas the lower one adhered very firmly and could not be washed off at all.

After washing, the slide was dried in a jet of dry hot air and submerged again in the colloidal solution. Then the whole procedure was repeated until a sufficient number of layers had collected on the chromium-plated metal slide to give the same interference colours, *i. e.* yellowish-brown, dark-blue, light-blue, yellow, red, *etc.*, as the polylayer films of calcium stearate.

Knowing the number of layers (attached during submersion) of the colloid hydroxide that give the same colour as the films of calcium stearate, one can compute the thickness of the films of the colloidal hydroxide with the formula employed previously by the author ([5]) in an investigation of copper hydroxide films:

$$t_2 = t_1 \frac{n_1}{n_2} \qquad (1)$$

where t_1 is the thickness of the polylayer film of calcium stearate having the corresponding colour; t_2 is the thickness of the polylayer film of the colloidal hydroxide having the same colour as the film of calcium stearate; and n_1 and n_2 are the indices of refraction of light for the stearate and the hydroxide.

Employing this formula, we obtained the values for the thicknesses of polylayer films of iron hydroxide given in Table 2.

Table 2

Colour and Thickness of Polylayer Films of Colloidal Iron Hydroxide

Colour of film	Thickness of film (in Å)
Yellowish-brown	298.5
Dark-blue	582.1
Light-blue	867
Yellow	1151
Red	1435
Blue	1720
Green	2004
Reddish-yellow	2289

From the thickness of a polylayer film and the number of submersions it is possible to compute the thickness of a unilayer, monomicellar film of iron hydroxide.

As the colloidal solution aged, the film on its surface became more and more compact and the computed film thickness became constant. In the experiments

with iron hydroxide the computed film thickness reached a constant value after several days (10 to 12 days), namely 37.3 Å or about 4 mμ. Determinations of the dimensions of iron hydroxide particles by ultrafiltration ([6]) and other methods ([7]) gave values ranging between 20 and 40 mμ. Comparing this quantity with the value of 4 mμ for the film thickness, it is easy to see that colloidal particles of iron hydroxide should have the shape of a thin plate, as has been shown in a number of other investigations ([8]).

Thus, the investigation of the films that form spontaneously on the surfaces of hydrosols during their ageing provides a new method of determining the dimensions and shape of colloid particles.

Laboratory of Colloidal Chemistry.
Institute of Chemistry.
Urals Branch
of the Academy of Sciences of the USSR.

Received
7.XII.1944.

REFERENCES

[1] С. Г. Мокрушин, Бюлл. Всес. хим. о-ва им. Д. И. Менделеева, No. 7, 38 (1941). [2] A. Frumkin, S. Reichstein and R. Kulvarskaja, Kolloid-Z., **40**, 9 (1926); T. Alty, Proc. Roy. Soc., A. **106**, 315 (1924); T. Alty and B. W. Curie, ibid., **122**, 622 (1929). [3] K. Blodgett, J. Am. Chem. Soc., **57**, 1007 (1935). [4] C. G. Mokrushin, Kolloid-Z., **70**, 48 (1935); ЖФХ, **5**, 1082 (1934). [5] C. G. Mokrushin and P. S. Konyayev, Kolloid-Z., **77**, 301 (1936); ЖФХ, **5** (1), 95 (1935). [6] J. Duclaux et M. Amat, J. Chem. Phys., **35**, 379 (1938); M. Amat et J. Duclaux, ibid., **36**, 256 (1939). [7] Ф. Ган, Дисперсионный анализ, стр. 184, 1940. [8] Cotton et Mouton, C. R., **141**, 317 (1905).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

AGROCHEMISTRY

Pb CONTENT IN DIFFERENT PARTS OF THE CARYOPSIS OF SPRING-WHEAT AND IN THE PLANT AS A WHOLE

By M. M. OKUNZOV

(Communicated by A. A. Richter, Member of the Academy, 5.VIII.1944)

Along the rest of minor elements lead is invariably found to be present in the ashes of animal and vegetable organisms, and its influence upon the latter is a beneficial one.

According to Vinogradov ([2]), the concentration of Pb in different organisms may vary within a range of 10^{-3} to 10^{-4} per cent to live weight. The favourable influence of lead upon various organisms has repeatedly been pointed out in literature ([4], [5]). The author of the present note has also succeeded in demonstrating that the effect of lead, if introduced into caryopses in very small amounts, is a positive one, like that of some other metals ([3]).

In the present work the author was interested in the question of the penetration of Pb into the caryopsis of wheat and into the plant as a whole, and in the way the element is distributed among the various parts of the caryopsis.

Method. As a material for the work use was made of the spring-wheat *Caesium* 0111. Lead was taken in the form of its nitrate, at a concentration of 0.01 *N*; it was introduced into the caryopsis of wheat in two ways. According to the first method, the caryopses were allowed to soak during 48 hours in a solution of Pb nitrate, which was in excess to the swelling capacity of the material. According to the second method, the caryopses were allowed to swell to no more than 45 per cent of their moisture capacity. The procedure was the same as applied in the case of vernalization and has been described in detail in our earlier paper ([3]). In the subsequent this method will be termed the «stimulation», while the other method will be called the «swelling» of the caryopses.

Upon stimulation and swelling the caryopses were twice rinced in distilled water. The concentration of lead was determined in the coat, endosperm and germinating embryo separately.

The plants were grown on filtering paper whose border was immersed into distilled water. To the analyses were subjected germinated embryos, seven-day-old, together with their root system, while the remains of the caryopses were removed.

On account of the very small quantities of Pb which penetrate into the caryopsis and the plant, we availed ourselves of the radioactive method of lead determination ([1]), which is the most sensitive and by means of which Pb concentration can be determined with an accuracy up to the seventh decimal sign. This method consists in contaminating common lead with its radioactive isotope, RaD, possessing the same properties as the common lead. When moving in a medium, inactive lead is invariably accompanied with its radioactive isotope, the quantitative relation between the two being constant. Owing to this, it was possible to measure the quantity of inactive lead according to the intensity of radioactive decay.

In the work here described nitrate of inactive lead was added its radioactive isotope, RaD, the concentration of the former being brought to 0.01 *N*. The solution obtained was found to contain 0.01036 g of $Pb^{\cdot\cdot}$ per 10 cm^3, the intensity of radioactive decay being equal to 26.032 fissions per minute. Accordingly, knowing the intensity of radioactive decay of RaD contained in the weighed portion under test, it is no difficult matter to find the amount of inactive $Pb^{\cdot\cdot}$ it contains.

The intensity of radioactive decay was measured with the aid of an alpha-electroscope. It was evaluated according to the number of β-particles, for which purpose a tinfoil screen had to be mounted. It is quite evident that the procedure permitted of determining only the lead that had been absorbed by the plant and its caryopses.

The seeds were treated by the two methods described above with a $RaD + Pb^{\cdot\cdot}(NO_3)_2$ solution, and also a RaD solution.

Table 1

Pb Concentration in Various Parts of Caryopses Stimulated with a $RaD + Pb^{\cdot\cdot}(NO_3)_2$ 0.01 *N* Solution

Experimental runs / Test-objects (from 150 embryos)	In dry matter					In ashes			
	Dry matter in g	Intensity of radioactive decay	Amount of $Pb^{\cdot\cdot}$ in g	$Pb^{\cdot\cdot}$ content in % to weighed portion	$Pb^{\cdot\cdot}$ content in % to total amount of lead	Weight of ashes in g	Intensity of radioactive decay	Amount of $Pb^{\cdot\cdot}$ in g	$Pb^{\cdot\cdot}$ content in % to weighed portion
Embryos	0.2052	0.253	0.0001007	0.04907	6.88	0.0154	0.253	0.0001007	0.65390
Endosperm . . .	2.4464	0.146	0.0000581	0.00237	3.94	0.0136	0.146	0.0000581	0.42721
Seed-coats	1.5616	3.304	0.0013149	0.08420	89.23	0.0718	3.304	0.0013149	1.83134
Total (in 150 whole caryopses) . .	4.2132	3.703	0.0014737	0.03498	100.00	0.1008	3.703	0.0014737	1.46200

Experimental Data. From Table 1 it should be inferred that, by stimulating the seeds, Pb is made to penetrate through the seed-coat into the embryo and the endosperm. If we are to judge by the content of lead both in the dry substance and in ashes, the largest amount of lead is contained in the coat, next to it follows the embryo, while the endosperm is the lowest in this element.

Table 2

Pb Concentration in Various Parts of Caryopses Soaked and Made to Swell in a $RaD + Pb^{\cdot\cdot}(NO_3)_2$ 0.01 *N* Solution

Experimental runs / Test-objects (from 150 caryopses)	In dry matter					In ashes			
	Dry matter in g	Intensity of radioactive decay	Amount of $Pb^{\cdot\cdot}$ in g	$Pb^{\cdot\cdot}$ content in % to weighed portion	$Pb^{\cdot\cdot}$ content in % to total amount of lead	Weight of ashes in g	Intensity of radioactive decay	Amount of $Pb^{\cdot\cdot}$ in g	$Pb^{\cdot\cdot}$ content in % to weighed portion
Embryos	0.2562	0.677	0.0002694	0.10515	3.89	0.0190	0.677	0.0002694	1.41789
Endosperm . . .	2.8578	0.289	0.0001150	0.00402	1.66	0.0136	0.289	0.0001150	0.84559
Seed-coats . . .	1.5048	16.440	0.0065426	0.43472	94.45	0.766	16.440	0.0065426	8.54125
Total (in 150 whole caryopses) . .	4.6188	17.406	0.0069270	0.14998	100.00	0.1092	17.406	0.0069270	6.34341

From the data of Table 2 it is seen that in caryopses made to swell in the solution, the distribution of Pb is similar to that observed in stimulated caryopses, *viz.* the highest concentration is found in the seed-coat, a somewhat lower, in the embryo, while the endosperm is found to contain the least amount of the element. After a 48-hour-long soaking in an excess of lead solution the caryopses were found to have absorbed four times as much lead as they did in the case of stimulation.

Consequently, the value of the method of chemical stimulation (or limited swelling) consists in that it makes it possible to introduce into the seeds definite quantities of minor elements, whereas in the case of swelling the amount of the salts supplied varies according to the period of treatment.

The possibility of a rigorous control of the amount of elements supplied to the seed is of prime importance, for if supplied in amounts exceeding a definite dose they may produce a detrimental effect upon the plant. Still more important it is in the case of ultra-microelements.

Table 3

Concentration of Lead in Whole Plants

Experimental runs	In dry matter					In ashes				
	Dry matter in g	Intensity of radioactive decay per weighed portion	Intensity of radioactive decay per 1 g	Content of Pb in g per weighed portion	Pb content in %	Weight of ashes	Intensity of radioactive decay per weighed portion	Intensity of radioactive decay per 1 g	Pb content in g per weighed portion	Pb content in %
Stimulation of seeds with RaD+ +$Pb^{**}(NO_3)_2$ solution	5.4930	0.712	0.130	0.0002833	0.00516	0.1698	0.712	4.193	0.0002833	0.16684
Seeds made to swell in RaD+ +$Pb^{**}(NO_3)_2$ solution	5.7242	2.567	0.448	0.0010216	0.01785	0.1686	2.567	15.225	0.0010216	0.60593
Seeds stimulated with RaD solution	5.4674	0.719	0.131	—	—	0.1664	0.719	4.321	—	—

The data of Table 3 make it evident that from the caryopses Pb penetrates into the plants. The concentration of Pb in such plants as were obtained from caryopses soaked in Pb solution was found to be 3.5 times higher than in plants that had been merely stimulated with the same solution.

The plants stimulated with a RaD solution without Pb or with it showed similar intensities of radioactive decay.

Department of Plant Physiology and Biochemistry.
The V. V. Kuibyshev State University.
Tomsk.

Received
5. VIII. 1944.

REFERENCES

[1] Е. С. Бугаков и В. Д. Нескучаев, Диффузия металлов, 1937. [2] А. П. Виноградов, Природа, 8—9, 28 (1933). [3] М. М. Окунцов, Тр. Биол. ин-та Томск. гос. ун-та, V (1938). [4] I. Stoklasa, C. R., **156**, 153 (1913). [5] A. Stutzer, J. Landw., **64**, 1 (1916).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

BIOCHEMISTRY

AN ATTEMPT TO STUDY METHODS FOR CULTIVATING GLADIOLI AS A NEW RAW STUFF FOR THE PRODUCTION OF VITAMIN C

By N. A. MAXIMOV, Corresponding Member of the Academy,
G. V. RAKITIN and R. KH. TUREZKAJA

It was pointed out several years ago that the leaves of gladiolus which is known as a pretty ornamental plant are rich in vitamin C. Thus, for example, according to Giroud (1) fresh leaves of this plant contain up to 0.496 per cent of vitamin C (the weight of the fresh green mass put at 100). In 1942 Shmuck (2) showed that certain varieties of gladiolus contain in their fresh leaves as much as 1.305 per cent of vitamin C.

Following the suggestion of the Central Administration of the Vitamin Industry, the Institute of Plant Physiology of the Academy of Sciences of the USSR undertook extensive investigation on gladiolus in the spring of 1943. The following work had to be done: in Moscow, by Dr. M. G. V. Rakitin and R. Kh. Turezkaja; in Erevan, by Prof. M. Ch. Čajlachjan; in Tashkent, by G. S. Soikina; in Frunze (Kirghiz Republic), by A. N. Krenke. The work proceeded under the general supervision of N. A. Maximov, Corresponding Member of the Academy.

In the present paper are reported data obtained in Moscow in 1942—1943. The results obtained at other points proved analogous. Quantitative determination of vitamin C was done by the method of indophenol titration (2.6-dichlorphenolindophenol) under application of metaphosphoric acid (3).

Over 200 varieties of gladiolus were studied. About 40 among them have been found to contain from 1 to 1.7 per cent of vitamin C, *i. e.* from 1000 to 1700 mg per 100 g of fresh leaves (Table 1).

It may be seen from Table 1 that many of the varieties of gladiolus are several dozen times richer in ascorbic acid than lemons, and hold the third place after such rich sources of vitamin C, as fruits of the wild rose or walnut.

Determinations of the vitamin content in different organs of the plant have shown that the first place is held by the leaves, the second by the flowering stems, the third by the flowers, and the fourth by the bulbs. Thus, for example, according to analysis the leaves of the variety Foche contain (per 100 g of fresh leaves) 1190 mg of ascorbic acid, flowering stems contain 407 mg, flowers 268 mg, and bulbs 75 mg only.

The leaves and the stems are thus the best raw material.

Gladiolus is propagated by large mature bulbs and by the little bulbs (babies) formed at their base. Our experiments have shown that the leaves of plants grown from little bulbs do practically not differ in vitamin C content from those raised from large bulbs.

With the growth of the leaves the vitamin content of the latter continually increases attaining its maximum towards the moment of its termination, which

Table 1

Vitamin C Content in Leaves of 20 Selected Varieties of Gladiolus (at the time of mass blooming)

Name or number of variety	Vitamin C content in mg per 100 g of fresh leaves	Name or number of variety	Vitamin C content in mg per 100 g of fresh leaves
Lord Pendelton	1700	Blue Bird	1270
Golden West	1600	No. 204 of the list of the Botanical Garden of the University of Moscow	1270
No. 101 of the list of the Botanical Garden of the University of Moscow	1550	Souvenir	1260
Red Emperor	1500	Baron Jules Hulot	1250
Giant Nymph	1400	Rose Laurouch	1250
Deutschland	1350	Luisa Cander	1220
No. 201 of the list of the Botanical Garden of the University of Moscow	1350	Lilac Wonder	1200
No. 242, same	1330	Violet superba	1200
Frank Pendelton	1330	No. 102 of the list of the Botanical Garden of the University of Moscow	1200
Red Canna	1280	No. 203, same	1200

coincides with the period of mass blooming. During the next period, prior to the autumn frosts, the vitamin content remains nearly constant and begins to decrease along with the yellowing of the leaves (Table 2).

Table 2

Vitamin C Content in Leaves of Different Age (mgs per 100 g of fresh weight)

Variety	Time of the first appearance of flowering shoot	Period of mass blooming	Beginning of yellowing of leaves
Golden West	994	1605	1271
Frank Pendelton	850	1310	1092
Red Emperor	1265	1525	1213
Maiden's Blush	703	995	891
Flaming Sword	668	907	775
Foche	737	1120	981
Halley	665	1170	900

In Table 3 are presented some data on the dynamics of increment of the total mass of leaves and stems.

It can be observed from Table 3 that the green mass grows during the last fortnights of the growing season, its increment being a comparatively slight one in the case of plants grown from large bulbs (from 10 to 11 g), while it is very considerable in the case of plants originating from «babies» weighing 0.8 to 0.9 g.

It is easy to see from Tables 2 and 3 that gladioli grown from large bulbs are to be harvested towards the end of the mass blooming, while in the case of plants grown from «babies» the harvest should begin 2.5—3 weeks later. Earlier harvesting is undesirable lest the young bulbs fail to reach maturity and suffer during winter storage.

Table 3

Average Weight of Green Mass of Gladioli at the Period from the 16th of August to the 25th of September (in g)

Date of sampling	Variety Maiden's Blush		Variety Faust	
	Plants from bulbs weighing from 10 to 11 g	Plants from bulbs weighing from 0.8 to 0.9 g	Plants from bulbs weighing from 10 to 11 g	Plants from bulbs weighing from 0.8 to 0.9 g
16.VIII	32.5	8.4	30.0	5.6
31.VIII	35.0	9.0	32.5	6.3
15.IX	35.6	13.5	33.2	13.4
25.IX	36.2	14.5	35.0	14.0

The yield of the green mass is strongly influenced by the feeding area (Table 4).

Table 4

Yield of Green Mass in Gladiolus as Affected by the Size of the Feeding Area

Plants grown from large bulbs			Plants grown from little bulbs		
Distance between rows in cm	Distance between plants in rows in cm	Average weight of green mass of a single plant in g	Distance between rows in cm	Distance between plants in rows in cm	Average weight of green mass of a single plant in g
Variety Halley					
40	25	48.5	20	15	7.4
30	20	46.2	15	10	5.2
20	15	45.2	10	5	4.3
15	10	38.1	5	5	2.5
10	5	30.3			
Variety Type					
40	25	33.6	20	15	4.4
30	20	29.1	15	10	3.9
20	15	27.2	10	5	3.6
15	10	25.3	5	5	2.6
10	5	23.0			

The greater the area of feeding, the greater is the yield of the green mass of a plant. However, the total yield of the green mass per unit area depends not only upon the area of feeding of the individual plants, or rather not so much on it, as on the number of plants per unit area.

A correct solution of the problem as to the number of plants per unit area is the following: the plants should be distributed in such a way, as to secure the possibility of rational working conditions (planting, cultivation, harvesting).

We arrived at a conclusion that the best solution is attained by the sowing in broad strips; for large bulbs two-row strips should be preferred, 50 cm apart, the two rows being 25 cm apart and the bulbs planted at 10 cm intervals; for the little bulbs, four-row strips should be adopted, 50 cm apart, the rows being 10 cm apart and the little bulbs set at intervals of 5 cm.

One hectare would thus take about 270 000 large bulbs (or about 3 tons), while it would take about 1 000 000 little bulbs (or 0.5 ton).

The depth of planting of large bulbs should be as follows: with light soils about 15 cm (from the surface of the soil to the base of the bulb); with medium soils, about 12 cm; and with heavy soils, about 10 cm. In the case of young bulbs the corresponding figures would be 6, 4 and 2 cm.

Planting should be done as early as possible; light and short frost are not dangerous for the sprouts. Harvesting of the green mass should be done after blooming is over. This makes it possible to make good use of the fine flowers of the plants. The bulbs are to be lifted after the green mass is cut.

When large bulbs are planted, one hectare yields about 8 tons of the green mass (5.3 tons of leaves and 2.7 tons of stems); in case of little bulbs the yield would come to about 6 tons (5.4 tons of leaves and 0.6 ton of stems).

With a vitamin C content averaging 1.5 per cent (of the fresh mass) in the leaves and 0.5 per cent in the stems the yield of the green mass from an area of 1 hectare would give about 93 kg or 1.86×10^6 human doses of vitamin C (daily dose amounts to 50 mg). When small bulbs are planted, the yield of the vitamin will amount to 84 kg or 1.68×10^6 human doses. In the course of storage and working up, a certain fraction of the vitamin will naturally be lost, so that the actual yield would be lower. However, the figures presented above give a fair idea of the possibility opened for the vitamin industry by the cultivation of gladioli.

Gladiolus has a high coefficient of propagation. Every hectare of area planted with large bulbs yields on the average about 400 000 of new bulbs and about 5 000 000 little bulbs. This amount of material is sufficient for planting about 6.5 hectares. Plantations of the little bulbs are still more productive. Every hectare yields in this case about 1 000 000 of large bulbs and about 5 300 000 of little ones. This number would suffice to plant 10.2 hectares.

One should finally point out that, apart from raw material for vitamin production, plantations of gladioli yield enormous quantities of beautiful flowers which are very popular in this country.

The K. A. Timiriazev Institute of Plant Physiology,
Academy of Sciences of the USSR.

Received
5. X. 1944.

REFERENCES

[1] A. Giroud, Protoplasma-Monographien, **16** (1938). [2] А. А. Шмук, Доклады ВАСХНИЛ, 11—12, 10 (1942). [3] D. Glick, Z. f. Physiol. Chem., **245**, 211 (1936).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

GEOLOGY

SEDIMENTATION, DENUDATION AND CHANGES IN THE ORGANIC WORLD

By B. L. LICHKOV

(Communicated by W. I. Vernadsky, Member of the Academy, 28.VII.1944)

The illuvial mode of sedimentation at the continents and at the sea bottom by way of percolation of aqueous solutions through subaerial soils, marine silts and analogous superficial formations—peats, turf ores, vegetable masses, *etc.* (1), is as natural a mode of deposition of sediments as the purely mechanical one. If this be considered, illuvial sedimentation can be said to be responsible for the formation of sedimentary new rocks in both above named media—the subaerial and the marine one.

The essence of the process which takes place during illuvial sedimentation is that soils, marine silts and their analogues by means of the waters penetrating from the surface downward, transmit their sol elements to the rocks. It should be borne in mind that the stock of these elements in the soil mantle, marine silts and their analogues on the surface of the lithosphere is no unlimited one; in order that the supply of rocks with these elements suffers no break in the process of sedimentation, they should be continuously supplemented. This is brought about by the denudation process (1). By centres of denudation we understand such areas of land from which the water denudational streams carry to the remaining territory of the land and to the seas the products of weathering which lie exposed on the surface at these centres; in the first place this refers to the eluvium of rocks.

At these phases of the Earth's life when denudation went on on a wider scale, the general centres of the Earth's denudation were distinctly pronounced. These were the polar-glacial regions of continental lowlands and mountainous areas covered with ice caps. It is known that around continental-polar centres of denudation, as a result of the work of denudation waters, accumulation belts were situated in crowns, occupying vast territories, whole continents; the vast area of these accumulation regions alone is evidence of the scale of work of the polar centres of denudation. No less considerable was the denudation work of the synchronous mountainous glaciations of the centres of denudation, judging from the belts or zones of accumulations produced near them. In the post-glacial time the situation changed; because of the amount of waters which decreased along with the disappearance of the glaciers, and of the resulting decrease in the area of denudation, these accumulation belts ceased to be formed. During this phase the general centres of denudation became local centres, which was a consequence of the increase in the denudational work of waters, which at that time could no more produce whole large continental and piedmont zones of accumulation. This being considered, the following scheme of the history of denudation during the phases of the geological cycle may be suggested.

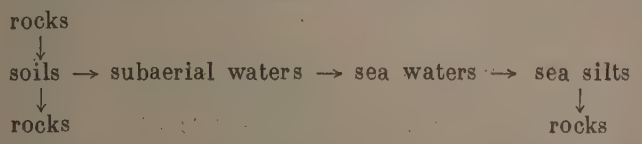
In the beginning of the cycle, at the phase of the maximum elevation of mountains and of the polar-continental land (the glacial epoch) the general centres

of denudation were distinctly pronounced. Later on, they were superseded by local centres of denudation. This corresponded to the phase of temperate climate which set in owing to the liberation of the polar land and mountain heights from ice. After the polar territories had become fully freed from ice, and the mountain heights had reached a minimum and had also become freed from ice, the local centres of denudation, gradually decreasing, were also obliterated: owing to a decrease in the quantity of waters on the Earth, the denudation fell nearly to zero there.

The xerophytic phase set in. The three phases of the geological cycle: the glacial phase, the phase of temperate climate and the xerophytic phase, distinguished according to R. S. Ilyin, are closely related to changes in the relief.

The phase of xerophytization came to an end after the epeirogenic elevation of the next cycle established new general centres of denudation, and the evolution began anew.

The process of supplying soils, sea silts and the analogues of both with sol elements, which the soil mantle transmits to rocks in the course of illuvial sedimentation, goes on according to the following scheme:



This scheme supplements an analogous scheme of W. I. Vernadsky ([2]), which did not reflect the rôle of denudation and did not take into consideration the rôle played by soils and their subaerial analogues in supplying the rocks. The fact that the rocks which in this scheme supply in turn the soil mantle, are raised above the soils, shows that they played their supplying rôle only in the process of denudation, the regulating rôle in this process being played by epeirogenesis. Along with the above-outlined history of denudation in the course of the development of the geological cycle, this fact leads one to the conclusion that throughout the development of the cycle, as the quantity of waters was diminishing and the area of denudation was decreasing too, the supply of soils with sol elements must have diminished as well.

In the glacial phase of the cycle the supply of sol elements was very large, owing to which the illuvial sedimentation was very intense, too. Further on, at the phase of temperate climate, both processes gradually decreased as the general centres of denudation were superseded by local centres; in this connexion the supply of soils with sol fall down to zero by the end of the cycle, *i. e.* in the xerophytic phase.

The change in the phases of the cycle from the glacial through the temperate to the xerophytic could not otherwise than affect in the first place the vegetation of the territory of the land and of the near-shore parts of the seas on the Earth's surface.

Waters and moisture being very abundant at the phase of the maximum denudation, in spite of the relatively severe climatic conditions in the form not only of zonal continental and mountain coolings, but of a general cooling, the vegetation must have been extremely profuse and variegated, with a prevalent part being played by forests. The phase of temperate climate is one of an extensive development of steppes and their struggle with forests under conditions of a gradual warming of the climate and a decrease of the amount of waters; by the end of this phase the area of development of forests gradually diminished, being substituted by steppes. Finally, the xerophytic phase is characterized by a further warming and a further decrease in the amount of waters; semi-arid regions and deserts which had existed in the preceding phases but had been then strictly localized, extended now over vast territories, as a result of which forests almost disappeared, and semi-arid and arid formations became the predominating one, steppes being preserved along with them.

It is evident that each vegetable formation—forest, steppe, desert—is not only a floristic but also a faunistic complex, from which it follows that a change in a vegetable formation is inevitably accompanied with a change in the complexes of the animal world.

An insufficient quantity of waters, a relief but slightly dissected and a warm climate are the characteristic conditions of the xerophytic, arid phase. Their consequence was a minimum removal by waters, and a supply to the soils of sol elements of the soil cover; instead of these, soluble salts were removed, which—owing to small quantity of waters and the general absence of drainage—were retained on the land, causing a general increase in the amount of salts in the soils as well as their local concentrations in the form of alkaline soils, solonetz, *etc.* Excessive concentrations of salts in the soil mantle of this phase and an insufficient amount of waters restrict the possibilities of the development of vegetation on the Earth: both on the land and in seas, the distribution of vegetation and the abundance of individuals are reduced, so that this phase may be regarded as a crisis of the vegetable kingdom.

This could not but affect the animal kingdom, either.

On the land this phenomenon was due, on the one hand, to the soluble salts in the soil mantle reaching a concentration toxic for the plants; it was brought above by a drying of the soils caused by water deficiency. In the seas abnormal conditions for individual growth and development of living organisms were created, the supply of salts being insufficient during this phase; because of the poor drainage of the relief, the salts were retained by the land. The given phase of the cycle was therefore a phase of great extinctions — disappearance and reduction of entire orders and classes of animals and plants.

Being confined to the end of the cycle, this phase of great extinction preceded the elevation of mountains at the beginning of the next cycle. Empiric data accumulated by classical geology, bearing on the history of the organic world, fully confirm this regularity: the «great Mesozoic extinction» during which the larger part of reptiles, ammonites, belemnites, *etc.* perished, took place on the eve of the New Cymmerian elevations; similar great extinctions of mammals preceded the Alpine elevations; amphibiae vanished in large numbers in the Jurassic, before the Old Cymmerian vertical movements. All these cases of disappearance, as pointed out above, were related to a lack of food supply and an insufficient amount of waters for the terrestrial organic world in the xerophytic phases of the cycle.

Aside from the disappearance of large orders, classes, families, *etc.*, during the same phases those new types came into being which in the next cycle were to gain the field in the struggle for life. The course and the results of the struggle for existence in the organic world during the xerophytic phases of the cycle thus determined the peculiarities of the organic world of the next cycle. As distinguished from the xerophytic phase, the glacial phase was primarily one of great shifts in the organic world, in connexion with the formation of glaciers, their advances and retreats; this was an epoch of migrations accompanied with the disappearance of definite genera and species. After the situation had become more or less stabilized, and all the changes had acquired a gradual character, the phase of temperate climate was for the organic world the phase of an intense formation of species, which continued into the xerophytic phase.

From all the aforesaid the following conclusions can be made:

1. The illuvial sedimentation on land and in seas, being a part of the general process of sedimentation, depends to the same degree as the mechanical sedimentation on the process of denudation which attains its maximum value when a high relief with maximum contrasts is developed, the highest points of the relief exceeding the permanent snow-line. — which produces universal centres of denudation, waters being abundant at the beginning of the new cycle.

2. The height of the relief and its contrasting nature being determined by the movements of the Earth's crust, it may be said that the area of denudation is determined by synchronous powerful movements of the Earth's crust.

3. On the scale and intensity of denudation within a definite period of time as combined with the illuvial lithogenesis, are dependent the food supplies and the fresh waters which at a given time the Earth's surface may offer to the organic of the dry land.

4. The quantity of the food supplies in the seas depends on the same conditions.

5. The end of the cycle is a phase of starvation for the whole organic world of the Earth; and for the terrestrial life it is at the same time a phase of water deficiency.

6. All the above being considered, in particular the regular periodicity of the advent of starvation phases at the end of the geological cycle and their being actually confined to those parts of the geological periods during which large groups of living beings—classes, orders, *etc.*—disappeared, we are justified in saying that the cause of these extinction were not various climatic changes, as it has been considered by N. N. Yakovlev, D. N. Sobolev, M. V. Pavlova and others, but more strictly speaking,—changes in the amounts of food supplies and of land waters.

Geological Institute.
Tadjikian Branch of the
Academy of Sciences of the USSR.

Received
28. VII. 1944.

REFERENCES

[1] B. L. Lichkov, C. R. Acad. Sci. URSS, XLI, No. 4 (1943). [2] W. Vernadsky, IV Confér. Intern. de Pédologie, Rome, p. 4, 1926.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

MINERALOGY

LAZULITE FROM A QUARTZ VEIN IN THE REGION OF MOUNT SURA-IZ IN THE NEAR-POLAR URALS

By M. F. BELIAKOV

(Communicated by A.A.Polkanov, Member of the Academy, 5.VII. 1944)

In the USSR lazulite occurrences are known in «secondary quartzites» of Kazakhstan ([7-9]), in andalusite-sericite-quartz rocks of Armenia ([5,6]) and in pegmatite formations of Eastern Siberia ([12, 14, 15]). In Kazakhstan and Armenia lazulite is associated with deposits of andalusite; in Eastern Siberia, with mica occurrences. Almost in all these places lazulite is found in small quantities, occasionally in the form of microscopic grains, and it is only in Kazakhstan, in the Chok-Part as massif, that it attains a relatively high concentration. It should be noted that lazulite has never been recorded from quartz veins of this country, while in other countries it is in quartz veins that this mineral occurs the most frequently.

Now a new place of lazulite occurrences in the USSR can be added to the small number of those already known, *viz.* the Near-Polar Urals. The mineral has been found on the western slope of the Ural Range, on Mount Starik, near the well-known deposit of rock crystal, the Sura-Iz. The area is composed of metamorphic rocks of the pre-Cambrian and Lower Palaeozoic, highly schistose and broken by a system of joints. The most characteristic joints are: (1) conformable with the enclosing rocks (along the strike and the dip); (2) conformable along the strike only (along the dip making an angle of about 90° with the schistosity of the rocks).

Most of the quartz veins of the Sura-Iz region rich in minerals ([13]), like those of some other localities in this part of the Urals ([1, 2]), are confined to the second jointing; the quartz veins filling the first jointing are relatively lower in various minerals.

Lazulite has been found in a lenticular quartz vein lying conformably (both along the strike and along the dip) among Cambrian (?) sericite-quartzite schists, forming mutual transitions from massive quartzites to finely schistose rocks; there are typically metamorphic rocks formed at the expense of sedimentary deposits ([10, 11]). The enclosing rocks are curved around the vein in accordance with the shape of the quartz body. The vein strikes NE 30°, and dips to the NW at an angle of 62°; the thickness of the vein does not exceed 15 cm. The vein quartz in the general mass is milky-white, but some portions are water-transparent. It is broken by a network of small cracks intersecting at right angles, which imparts to the quartz a peculiar «schistosity». The presence of regular cracks in the vein quartz is explained by the author by post-mineral tectonic movements of a local nature*.

Lazulite is enclosed directly in the vein quartz and forms a number of small accurate bands and irregular accumulations. Paragenetically associated with

* Regular cracks of a similar kind were observed also in some conformable quartz veins in the Lapcha (1937) and Khobe-Yu (1938—39) deposits.

lazulite are: sericite—an invariable associate of lazulite, calcite (?), hematite and rock crystal. The crystals of calcite had dissolved during the change in the conditions of the hydrotherm and did not persist; instead of them the veins contain rhombohedral voids with outgrowths of quartz ribs, oriented along the cleavage parallel to the face (010). The growth of these quartz ribs evidently proceeded simultaneously with the crystallization of calcite, the process being essentially the same as the formation of «saw cuts» on the quartz crystals from Mount Sura-Iz(4).

Potash mica occurs both in the vein quartz, and especially in the lazulite itself, intergrowing with the latter. Hematite is present in quartz as micaceous hematite. Rock crystal is encountered in the form of individual underdeveloped crystals; they have usually no more than 2—3 faces of the prism, faces of rhombohedra being altogether absent; the width of the faces does not exceed 0.5 cm.

From its physical and chemical properties, the lazulite from the Near-Polar Urals is on the whole similar to its common varieties from other places. In the solid mass the mineral is represented as granular aggregates. Brittle. Colour, sky-blue. Lustre, vitreous. Hardness, 5—5.5. Specific gravity of the selected grains of lazulite 3.04; in a fine mixture with potash mica it is as low as 2.8. Streak, white. Upon heating the mineral becomes white, while the flame burns green; when moistened with cobalt nitrate, it regains its former colour. It is not soluble in acids and retains its natural colour. With soda it fuses well. Under the microscope, it shows hardly any pleochroism, which is somewhat uncommon for this mineral; from literary data, the lazulites from other places show a distinct pleochroism. The mineral is biaxial, optically negative, with an indistinctly pronounced cleavage passing parallel to the N_g axis, a birefringence of muscovite, and a positive relief. The refractive indices, determined by the immersion method, are 1.634 and 1.616 along N_g and N_p, respectively; $2V = 67°$.

An analysis of the selected grains of lazulite made by the author at the Chemical Laboratory, Department of Petrography of the Leningrad State University, is given below (in per cent):

MgO	12.54
FeO	2.08
CaO	none
Al_2O_3	32.99
P_2O_5	46.38
Hygrosc.	0.27
Loss on ignition	6.05
Total	100.31

Both from its optical constants and chemical composition the lazulite here described approaches the lazulite from the Paraga-Chai andalusite deposit in Armenia (5,6).

Recalculating the analysis, we obtain the following formula of lazulite: $Al_2O_3 \cdot 1.01\ P_2O_5 \cdot 1.04\ (Fe'', Mg)\ O \cdot 1.08\ H_2O$.

The coefficients of the oxides may practically be put at unity; hence, after Dana (3), the formula of the mineral will be $2AlPO_4 \cdot (Fe'', Mg)\ (OH)_2$, with a $Fe'' : Mg$ ratio $= 1 : 11$. From the latter ratio and from the analysis it may be seen that in the lazulite from the Near-Polar Urals magnesium considerably prevails over iron. The same is true of the lazulite from Paraga-Chai, where $Fe'' : Mg = 1 : 16$. It should be mentioned in this connexion that in the lazulites from other countries Mg also prevails, as a rule, over Fe'', the following ratios of Fe'' to Mg being most common (after Dana) : 1 : 12; 1 : 6; 1 : 2; 2 : 3. In the mineral here described no calcium has been discovered, which is characteristic of the typical lazulite *; qualitative reactions for Cu and Mn gave also a negative result.

* Lazulites containing Ca (up to 3 per cent) are separated by some authors as a special variety—calcium lazulites (see Dana); however, such a separation is hardly justifiable.

Lazulite was formed at the high-temperature phase of the hydrothermal process by precipitation from aqueous solutions rich in phosphorus and alkalies of deep-seated origin (the geophase H—I of hydrothermalites after A. E. Fersman). The whole body of observations available enables us to consider that the source of the emanations of phosphorus fixed in the form of lazulite was a magnetic focus. The genetic relation of lazulite to a supply of phosphorus from the deep-seated melt has been indicated also by Nakovnik ([9]), Movsessjan ([5],[6]) and other authors; Nakovnik, in particular, refers its lazulite to the group of pneumatolytic minerals.

The invariable paragenesis of lazulite with potash mica (with sericite or muscovite) is of interest; lazulite is, evidently, capable to precipitate only if a sufficient quantity of alkalies is available.

Certain grains of the Uralian lazulite, owing to its colour and relative stability, can be used in jewellery.

Received
5. VII. 1944.

REFERENCES

[1] М. Ф. Беляков. Проблемы Арктики, № 9 (1940). [2] М. Ф. Беляков. Зап. Всерос. минер. общ., в. 4, 542 (1940). [3] Э. С. Дана. Описательная минералогия, стр. 296. 1937. [4] Г. Г. Леммлейн. Сборн. Приполярный Урал, сер. Уральск., в. 6, стр. 90, 1937. [5] S. A. Movsessjan, Bull. Acad. Sci. URSS, sér. géolog., № 6 (1939). [6] S. A. Movsessjan, C. R. Acad. Sci. URSS, XXXI, No. 1 (1941). [7] Н. И. Наковник, Пробл. сов. геологии, № 6, 236 (1933). [8] Н. И. Наковник. Тр. ЦНИГРИ, в. 58, 8 (1936). [9] Н. И. Наковник, Сов. геология, № 11, 43 (1938). [10] Н. А. Сирин и Г. В. Шмакова. Сб. Проблемы геологии Полярного Урала. Тр. Аркт. ин-та, 74, стр. 12, 49, 68, 78, 79 (1937). [11] Н. А. Сирин. Тр. ин-та геол. наук АН СССР, в. 40, петрограф. сер. (№ 13), 35 (1941). [12] А. Е. Ферсман. Пегматиты. I. стр. 220, 281, 534, 537, 1940. [13] М. Г. Хисамутдинов. Зап. Всерос. минер. общ., ч. 69, в. 1 (1940). [14] А. А. Якжин. Сб. Слюды СССР, стр. 268, 273, 276, 277, 1937. [15] А. А. Якжин. Тр. Вост.-Сиб. геол. треста, в. 22, стр. 52, 66, 102, 107 (1937).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

MINERALOGY

ON THE CLASSIFICATION OF PHOSPHORITES

By G. I. BUSHINSKY

(Communicated by A. N. Zavaritsky, Member of the Academy, 26.VI.1944)

In 1888 Penrose ([10]) suddivided all phosphorites into four groups: (1) amorphous nodular phosphorites; (2) beds of phosphate limestones; (3) guanos and (4) bone beds. In its main features this classification has been preserved until now and has only been worked out in more detail. The «phosphate limestone beds» have later on been termed «bedded phosphate rocks».

The first classification of Russian phosphorites was proposed by Engelhardt in 1891 ([8]). All the nodular phosphorites (bedded phosphate rocks were not known at that time) were subdivided by him into three types: (1) quartz-arenaceous, (2) glauconite-arenaceous, and (3) dense. The latter subsequently received the name of argillaceous. This classification fully satisfied the industrial requirements, for all of the Russian phosphorites could easily be made to fit within these groups. Quartz-arenaceous phosphorites were distinguished by their low P_2O_5 content and were difficult to grind; glauconite-arenaceous phosphorites were much more readily ground, and somewhat higher in phosphate, but owing to a high content of sesquioxides were not suitable for chemical treatment. Dense phosphorites were the most high-grade variety and could be used for the production of superphosphate.

Within the recent years much new material has been collected ([3–5], [9]) which permits of a more detailed classification.

The phosphate matter of phosphorites consists of minerals which have a crystalline lattice of the apatite type, *viz.* of fluorapatite, hydroxyapatite, carbonate-apatite-podolite and carbonate-fluorapatites ([3], [5], [9]).

To the latter belongs a group of minerals poorly known as to their structure, but extensively developed in nature, that of francolite (synonym—staffelite) and its cryptocrystalline variety—collophane, on the one hand, and kurskite, on the other. Each of these minerals is formed only under strictly definite geological conditions.

The so-called island phosphorites, developed on the islands of southern seas, are composed mainly of hydroxyapatite and in part of fluorapatite. These phosphorites have been formed on land as a result of guano decomposition. They have been but little studied as yet in point of mineralogy.

Composed of fluorapatite and carbonate-apatite (podolite or dallite) are the radial-nodular phosphorites of the Silurian of Podolia (Table 1) and of the Cretaceous deposits of Wyoming, USA. Both occur in clay shales of marine origin. The surface of these phosphorite concretions is smooth or tubercular, and in the centre they have a star-shaped cavity. The mineralogical distinctions of the radial-nodular phosphorites suggest their being separated out as an independent type.

There is no agreement in opinion as to the composition of the phosphate minerals which make up the bedded and concretionary or nodular phosphorites. The

optical properties (ε, ω; ω—ε) and X-ray data reveal no difference*. However, a large number of detailed chemical analyses carried out during the recent years, accompanied by petrographical research, have shown that the CO_2 content in concretionary phosphorites is in all cases higher than in bedded phosphorites and shows a definite ratio to P_2O_5 (Table 1). Relatively pure phosphorites are meant here, in which under the microscope no carbonates have been discovered.

Table 1

Content of Principal Components in Type Sea Phosphorites

Constituents	Bedded phosphorites			Concretionary phosphorites			Radial-nodular phosphorites Podolia	
	Volsk	Kara-Tau	Idaho, USA	Egorievsk	Briansk	Egorievsk*		
P_2O_5 . . .	34.82	33.33	32.24	24.86	19.99	31.39	36.18	36.81
CaO	50.07	46.64	45.96	39.33	29.82	49.55	49.79	49.65
CO_2	2.73	2.09	1.80	4.36	3.88	5.34	4.43	0.33
F	3.44	2.89	3.40	2.55	2.06	3.61	0.57	2.21
CO_2:P_2O_5	0.08	0.06	0.06	0.17	0.19	0.17	—	—

* Phosphatized wood from a Portlandian phosphorite bed.

In the classification of Smirnov (6) it may easily be seen that among readily assimilated are classed all the concretional phosphorites, and among the poorly assimilated, the bedded ones. Only the case of Algeria phosphorites has remained rather obscure. Experiments with oats grown on podzolized soil of the Dolgoprudny experiment field in 1926 showed that < 0.05 mm fractions of phosphorite flour from concretional phosphorites gave a yield of 17—22 g per vessel, and of that from bedded phosphorites, 8—10 g per vessel (7).

Table 2

Comparative Table of Peculiarities of Type Marine Phosphorites

Peculiarities	Types of phosphorites		
	Bedded	Concretionary	Radial-nodular
Phosphate mineral . .	Collophane	Kurskite	Dallite-apatite
CO_2 : P_2O_5 (by weight)	0.08±0.03	0.17±0.03	—
P_2O_5 content in phosphorite, %	Usually 30—37	Usually 10—30	36—37
Amount of admixtures	Usually little	Usually much	Very little
Oolitic structure . . .	Common	Rare	Absent
Organic remains . . .	Rare	Common	Very rare
Glauconite	Rare	Frequent	Absent
Dolomite	Common	Absent	»
Place of accumulation .	Usually geosynclines	Usually platforms	—
Thickness of beds . . .	Usually large (up to 17 m)	Usually small (up to 15 m)	Disseminated concretions
Degree of assimilation of raw phosphorite flour by plants . . .	Poorly assimilated	Easily assimilated	Poorly assimilated
Industrial working . .	Chemical treatment	Ground to flour	Chemical treatment

These facts, as well as a number of other peculiarities summed up in Table 2, make one suppose that collophane and kurskite are if not different species at any rate varieties of carbonate-fluorapatite. As a rule, concretionary phosphorites are not formed together with bedded ones and do not pass one into the other. The so-called phosphorite flagstones (Kursk, Aktiubinsk, Egorievsk, *etc.*) consist of phosphorite concretions cemented together with phosphate, and belong to concretionary phosphorites.

* Kurskite has usually a more high birefringence than staffelite.

The classification suggested by the author is based upon a genetic principle (Table 3). In the case of chemically precipitated phosphorites it proved impossible strictly to adhere to this principle, and one had to have recourse to differences in structure and texture. To these differences are related so many other important characters (Table 2) that in the course of a further study of phosphorites definite genetic factors will be certainly find out, which have been responsible for these characters and which it will be possible to make use of for genetic classification.

Table 3

Classification of Phosphorites

	Phosphorites		Examples of deposits	Phosphate mineral
Marine	Chemically precipitated	Bedded	Volsk, Khoper, Kara-Tau, Rocky Mts. (USA)	Collophane (francolite)
		Nodular (concretionary)	Egorievsk, Kursk, Aktiubinsk, South Carolina	Kurskite
		Concretional-nodular	Podolia, Gotland, Wyoming (USA)	Dallite-fluorapatite
	Organogenous	Obolus	Estonia	Collophane
		Bone	No independent deposits exist	Collophane-kurskite (?)
Continental	Organogenous	Bone	Eldar steppe	Hydroxyapatite-collophane
	Organogenous-metasomatic	Island	Nauru, Curaçao	Hydroxy-apatite, oxy-apatite, collophane and others
		Cave	Austria, South Africa	
	Metasomatic	—	Florida, hard rock Tennessee, white rock	Collophane
	Hydrothermal	—	Antonovo-Lipovskoye (Urals), Estremadura (Spain)	Francolite
Worked	Residual	—	Syzran (Batraki), South Carolina (USA)	Different
	Redeposited	—	Podolia, Florida, land pebble	Different
	Metamorphic	—	Teshik-Tas (Kara-Tau)	Fluorapatite

Further subdivisions of phosphorites may be conveniently made according to the composition of the admixtures, which has been attempted as for back as by Engelhardt ([8]) and continued by Arkhanguelsky ([1]).

Received
26.VI.1944.

REFERENCES

[1] А. Д. Архангельский, Фосфориты СССР, сб., стр. 23, 1927. [2] I. Borneman-Starinkevitch, C. R. Acad. Sci. URSS, XXVI, No. 8 (1940). [3] Г. И. Бушинский, Бюлл. Моск. общ. испыт. природы, отд. геол., **14** (2) (1936); **15** (5) (1937); **16**, (4) (1938). [4] Г. И. Бушинский, Агрономические руды СССР, **6** (1941). [5] А. В. Казаков, Тр. Научного ин-та по удобр., вып. 145 и 139 (1937). [6] Н. Д. Смирнов, Фосфоритование. сб., стр. 104, 1935. [7] Н. Д. Смирнов, Фосфорные удобрения и их качества, сб., стр. 221, 1938. [8] А. Н. Энгельгардт, Фосфориты и сидерация, 1891. [9] I. D. MacConnell and I. W. Gruner, Am. Mineralogist, **25**, 157 (1940). [10] R. Penrose, U. S. Geol. Surv., **46** (1888).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

MINERALOGY

RÔLE OF COLLOIDAL-DISPERSE MINERALS IN THE PROCESSES OF GROUND FREEZING

By I. SEDLETZKY and A. MALOVICHKO

(Communicated by V. A. Obruchev, Member of the Academy, 30.V. 1944)

1. It has been rather safely established by now that colloidal-disperse minerals play a decisive part in the determination of the physical and colloidal-chemical properties of loose rocks; the values of plasticity, volume capacity and other properties of clays, soils, loesses, are determined by the composition of the argillitites (1). The mineralogical composition of the fine fractions of clastollites is always taken into consideration when settling the question as to their utilization; for instance, the use of montmorillonite clays in the petroleum industry; of kaolin clays, in the ceramic industry, *etc.*

In the science of frozen grounds the mineralogical composition of friable rocks in which the processes of freezing and thawing of grounds are investigated, has up to the present been neglected, however.

On the basis of a study of colloidal-disperse minerals and of their importance in the physical-chemical processes taking place in grounds, the authors have advanced the hypothesis that the freezing processes largely depend on and are determined by the composition of colloidal-disperse minerals.

The experiments whose results are given below went fully to confirm this hypothesis.

2. As subjects of study three rocks of a different mechanical composition were used: sand, Beskudnikov loam and Jurassic clay. The amount of the fractions finer than 1 μ in the Beskudnikov loam and in the Jurassic clay was nearly similar (15 per cent and 17 per cent), while in the sand this fraction was altogether absent.

According to X-ray data, the composition of colloidal-disperse minerals in the Beskudnikov loam and Jurassic clay is different: the chief component in the Beskudnikov loam is kaolinite; in Jurassic clay, nontronite. The minor components are quartz, sericite and goethite in the Beskudnikov loam, and quartz in Jurassic clay.

Experiments in the freezing of the Beskudnikov loam and Jurassic clay have shown that while in the Beskudnikov loam predominated large fractures (ice-interlayers) located in the central part of the specimen, the Jurassic clay was characterized by the appearance of small undulatory cracks (ice interlayers) near the surface of the specimen.

In order to bear out the possible dependence of the different behaviour during the freezing of the Beskudnikov loam and the Jurassic clay upon the different composition of argillitites, experiments were carried out on pure minerals: kaolinite and nontronite.

3. X-ray photographs showing the changes in the internal structure of overmoistened kaolinite (w=65 per cent; the moisture capacity of this kaolinite w=50 per cent) at the three above stages (initial, frozen and subsequent to thawing) show that the initial condition of kaolinite is characterized by the presence on the X-ray photograph of a large number of water drops (dark spots). In the frozen condition kaolinite is characterized: by the presence of a large number of small undulatory horizontal cracks near the upper surface, the presence of a small number of very thick and very large cracks in the central and lower parts of the specimen. After the specimen of kaolinite is made to thaw, the small undulatory cracks disappear altogether.

The large cracks, while retaining their arrangement, lose their intensity, which may be accounted for by the fact that the moisture is re-absorbed by the ground.

The dark spots—water drops—fully retained their arrangement and shape on all the three X-ray photographs.

4. The same experiments with overmoistened specimens of nontronite showed results different from those obtained with kaolinite. In the frozen condition nontronite showed on the X-ray photograph the presence of a very large number of small undulatory cracks, parallel to the upper surface and filling nearly the whole of the specimen.

Large cracks were observed in a small number.

Thus, the picture of the changes of the internal structure of the specimens of pure minerals upon freezing is quite different for kaolinite and nontronite. The changes in kaolinite are similar to changes in the Beskudnikov loam, and the changes in nontronite resemble the picture observed in the Jurassic clay.

5. The experiments with kaolinite and nontronite with different moisture contents gave the same results.

Kaolinite with a moisture content approaching the total moisture capacity (w=45 per cent) shows the presence of a considerable number of water drops. In the frozen condition a small quantity of small and large cracks appeared.

In the thawed specimen the small cracks disappear, while the large ones are retained, but change their arrangement and intensity. It is of interest to note that the dark spots (drops) do not retain their arrangement, but are changing their place in the process of freezing and thawing. This confirms the view of Fedosov ([2]) as to the existence of strains originating in the grounds in the process of freezing, and of individual water inclusions migrating as a result of the work of those strains.

Besides, this experiment never resulted in the formation of cracks as generated by drops.

X-ray photographs of the initial state failed to reveal in any specimen of this kaolinite with a moisture content of 37 per cent, round dark spots due to water drops; a certain number of air inclusions (small dark spots of irregular shape) is observable instead.

In the frozen condition there are observed: a small number of small horizontal cracks near the upper surface; a certain (very small) number of large cracks, connecting the air interlayers and having a comparatively small size.

In the thawed condition the small horizontal cracks near the surface disappear; large fractures of the same size and at the same places as in the frozen condition are observable.

The latter is evidence of the fact that the processes of migration of water and air are less intensive in such thawing specimens as are lower in moisture.

An examination of the X-ray photographs of the same kaolinite with a still lower moisture content (w=27 per cent) shows that in the original specimen rounded moisture inclusions (drops) are completely absent, and that there is a certain number of voids filled with air.

In the frozen condition the specimen forms a very small number of fractures, all of them developing from voids which existed before freezing; most of these fractures are arranged horizontally, in the lower part of the specimen.

In the thawed condition a certain degradation of small cracks is observed, while there is no change in the large fractures (*i. e.* the process of migration which attends thawing is also one of small intensity).

6. Analogous X-ray photographs were also taken of nontronite with various moisture contents.

X-ray photographs, illustrating the alterations in the internal structure of the nontronite sample with a moisture content $w=85$ per cent, show that in the original sample large water inclusions are absent, while there is a certain number of large air inclusions.

In a frozen specimen there appears a strongly developed fracturing, characterized by the presence of small undulating cracks, parallel to the surface and filling almost the whole of the specimen, and by the presence of not very long, large fractures, filling the whole of the specimen, too.

In thawed condition the specimen retains the large fractures only, while the small ones, especially the fractures of the first type, disappear altogether.

The X-ray photographs illustrating the change in the internal structrue of the nontronite specimen with a moisture content $w=60$ per cent at the same stages show that in the initial state large water inclusions are absent, and there is a small number of voids and small cracks filled with air. In the frozen condition these voids develop into larger fractures, ice (air) interlayers. The fractures are arranged mainly horizontally. In the thawed condition the fractures degrade markedly, and like in all the preceding cases the smallest cracks and the smallest ramifications of the large fractures disappear.

Now if we are to compare frozen specimens of nontronite with frozen specimens of kaolinite, we observe that:

When the moisture content exceeds the total moisture capacity, the specimens form mainly fractures of the first type (small undulating), their number being considerably greater in nontronite than in kaolinite.

When the moisture content approaches the total moisture content or is somewhat lower, a maximum development of large fractures (of the second type) is observed; in kaolinite these fractures are longer and narrower; in nontronite, shorter and wider.

The number of small fractures decreases with a decrease in the moisture content.

If the moisture content is considerably below the total moisture capacity, no small cracks (of the first type) appear in the specimens, and only fractures of the second, or to say more correctly, of the third type are formed, which are associated with the presence of air inclusions.

Thus, the facts observed went to substantiate our assumptions as to the relation existing between the mode of fracture formation and the mineralogical composition of the ground. The nature of the fractures of the Jurassic clay, into which nontronite entered as the chief clay mineral, has been found to be almost the same as in fine nontronite. The nature of the fractures in the Beskudnikov loam has been found to approach that of the fractures in pure kaolinite, although not entirely.

If the moisture content corresponds to the maximum development of large fractures in the Beskudnikov loam, the fractures are still larger and thinner than in kaolinite.

The authors have carried out therefore supplementary experiments with such minerals as were also present in colloidal fractions of the Beskudnikov loam, *i. e.* with muscovite and quartz.

In view of the fact that very few of these minerals were available, the authors were obliged to confine themselves to the study at one moisture content only, *viz.* somewhat below the total moisture capacity. The tests were carried out on muscovite (mica) in the same three stages of the process. They have shown that muscovite in the frozen condition is also subject to the formation of fractures (*i. e.* that the migration follows the type of clays). As far as it can be judged from a single experiment, the nature of the fractures formed most closely approaches the one

observed in the Beskudnikov loam, *i. e.* a very small number of small cracks (of the first type) is observed at the very surface, while there is a comparatively large number of long thin fractures (of the second type). This also confirms our assumptions and explains why the nature of the fractures in the Beskudnikov loam is not quite similar to that of kaolinite.

It occupies an intermediate position between kaolinite and muscovite, since these two minerals are present there.

Moreover, the specimens of the two grounds (of Jurassic clay and of Beskudnikov loam) have been found to contain quartz. As should be expected, the migration of moisture in fine quartz was found to proceed after the type of sand and was not accompanied by the formation of fractures. This probably accounts for the fact that in the Jurassic clay and in the Beskudnikov loam, both of which contain a certain amount of quartz, the formation of fractures (especially of those of the first type) proceeds less intensely than in pure clay minerals.

Received
30. V. 1944.

REFERENCES

[1] I. Sedletzky and S. Yussupova, C. R. Acad. Sci. URSS, XXVI, No. 3 (1940).
[2] А. Е. Федосов. Физико-механические процессы в грунтах при их замерзании и оттаивании, М., 1935.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

CYTOLOGY

DEMONSTRATION OF PARANECROTIC CELL ALTERATIONS ON PERMANENT MICROSCOPIC PREPARATIONS

EXPERIMENTS ON *ALLIUM CEPA* AND *A. SATIVUM*

By P. V. MAKAROV

(Communicated by L. A. Orbeli, Member of the Academy, 15. VI. 1944)

One of the most characteristic features of the paranecrotic state of the cell is a very peculiar change in the structure and properties of the nucleus (1). These alterations were discovered either by vital staining (2), or ultramicroscopically (3). Aspects of paranecrosis could not yet be observed on ordinary histological preparations because of the coagulating effect of fixatives. My observations have shown that after fixation with osmic acid (OsO_4) and usual staining the nuclei retain their original microscopic aspect. Damaged cells are easily detected by the structurization appearing in them. It is possible, therefore, to «fix» in this way the paranecrotic cell alterations. This conclusion has been deduced from a material inaccessible to experiment (human tissues in wound complication). An experimental verification of the results obtained appeared to be of very material importance, however. With this aim in view I carried out experiments on the effect of various paranecrotic agents on the epidermal cells of *Allium Cepa* and *A. sativum*.

Method. Fragments of epidermis from the concave surface of the scales of bulbs of *Allium Cepa* and *A. sativum* were fixed before or after being subjected to various treatment in 0.25 per cent water (distilled) solution of OsO_4. To every 10 cm^3 of the fluid 1—2 drops of 0.5 per cent H_2O_2 were added to prevent osmic acid from being reduced by the tissue. After 24 hours the fragments were washed in water and stained with alum-carmin, haematoxylin after Böhmer plus eosin, or treated after Feulgen and mounted in Canada balsam. Parallel observations were made on living cells both in passing light and in dark field (paraboloid condensor).

Normal Condition. The structure of the normal nuclei largely depends on the qualities of the material used. If necessary precautions are taken, the nuclei look after fixation with OsO_4 like absolutely homogeneous vesicles without any traces of granular or reticular structure. Only nucleoli appear in them. Ordinary stains produce a perfectly diffuse coloration. Treatment after Feulgen has shown that the main chemical component of the nucleus, thymonucleic acid, is evenly distributed and forms no local condensations whatever, which could be taken to correspond to the nuclear skeleton (Fig. 1, *a*).

Analogous results have been obtained with ultramicroscopic observations. Nuclei appear to be optically empty in the dark field and cannot be distinguished from the dark cytoplasm. Microsomes engaged in Brownian movement vividly tinkle in the latter (Fig. 2,*a*). When viewed in passing light, the nuclei of living cells also appear to be homogeneous.

Folds are sometimes formed on the surface of the nucleus, which are easily discovered both *in vivo* and on preparations fixed with OsO_4.

It should be noted that nuclei of *Allium sativum* are denser and reacher in organic matter than those of *A. Cepa* (Fig. 1, *b*). Otherwise no differences between them can be observed.

Effect of Acetic Acid (CH_3COOH). After being kept during 30 minutes in weak CH_3COOH solution ($N/100$—$N/600$) and fixed in OsO_4, the

Fig. 1. Nuclei of epidermal cells of *Allium Cepa*. Fixation with 0.25 per cent osmic acid, treatment after Feulgen. *a*, normal condition; *b*, same for *A. sativum*; *c*, $N/200$ CH_3COOH 30 minutes; *d*, $N/200$ CH_3COOH 30 minutes, 1 per cent of glucose 1 hour; *e*, 3 *m* C_2H_5OH 30 minutes; *f*, 3 *m* C_2H_5OH 30 minutes, 1 per cent of glucose 1 hour; *g*, 50°C 10 minutes; *h*, mechanical influence; *i*, ultraviolet rays 10 minutes; *k*, same 5 minutes; *l*, same 5 minutes, 1 per cent of glucose 1 hour; *m*, 10 per cent of glucose 30 minutes; *n*, 10 per cent of glucose 30 minutes, 1 per cent of glucose 1 hour

nuclei show stainable reticular-granular structures. By varying the concentration of the acid it is possible to produce different degrees of the expression of these structures. When the damage is slight, granules are observed which evenly fill the nucleus. After a stronger treatment fusion of individual granules into reticular formations may be observed (Fig. 1, *c*). By means of vital observation in the dark field similar modifications in the inner nuclear structure can be detected (Fig. 2, *b*).

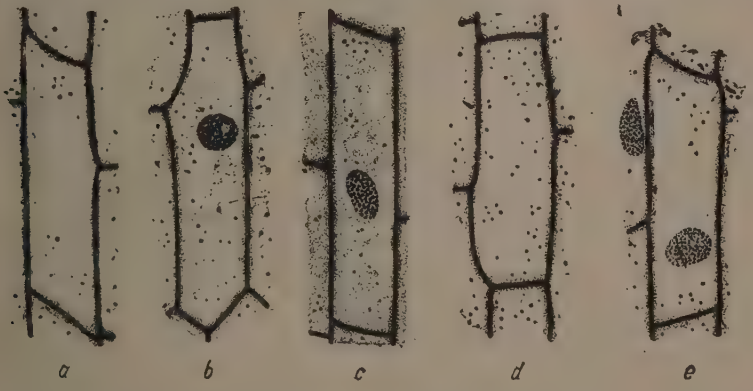


Fig. 2. Epidermal cells of *Allium Cepa* in dark field. *a*, normal state; *b*, $N/200$ CH_3COOH 30 minutes; *c*, 3 *m* C_2H_5OH 30 minutes; *d*, 3 *m* C_2H_5OH 30 minutes, 1 per cent of glucose 1 hour; *e*, 10 per cent of glucose 30 minutes

Optically empty nuclei seem to be filled with sparkling white granular substance, the cytoplasm is slightly opalescent. In other words, alterations are observed which are typical of paranecrosis ([3]).

The alterations described above (beginning with $N/200$ CH_3COOH) are reversible. When fragments of epidermis treated with the acid are transferred for 1 hour into pure water or 1 per cent glucose solution, they appear after fixation with OsO_4 again homogeneous (Fig. 1, *d*). When observed *in vivo* ultramicroscopically, they are optically empty.

Effect of Ethyl Alcohol (C_2H_5OH). Under the influence of 3—4 *m* of C_2H_5OH there appears a granular structure in the nuclei which may be clearly detected both on preparations fixed with OsO_4 (Fig. 1, *e*) and *in vivo* ultramicroscopically (Fig. 2,*c*). Under the influence of C_2H_5OH the cytoplasm observed in the dark field is found to produce opalescence, dark optically empty vacuoles appear in it and the microsomes become less brilliant. 2 *m* C_2H_5OH produce no visible alteration in the cellular structures. The damage produced by 3 *m* C_2H_5OH is easily repared, the nuclei assuming both ultramicroscopically and on permanent slides their homogeneous appearance (Fig. 1, *f* and Fig. 2, *d*). 4 *m* C_2H_5OH produce irreversible alterations.

Thermal Treatment. Heating to 40° C during 10 minutes produces scarcely any visible effect upon the cellular structure. When kept during a similar period of time at 50 and 60° C, a sharp structurization of the nuclei is observed. The clots which are formed are large and massive. Clear areas appear around the nucleoli (Fig. 1, *g*).

In conformity with literary data ([1]) it proved difficult to attain reversion after thermal treatment. Only in some of the cells which had been heated to 50° C could we observe reversion of the nuclei to their initial homogeneous state. In the majority of cases the cells were found to be dead.

Mechanical Damage. When pressure is exerted upon the fragments of epidermis near its centre, the nuclei in the zone of pressure appear on fixed slides to be structurized (Fig. 1,*h*). No reversibility could be observed. It should be noted that a similar effect is caused by cutting the cells. This accounts for the presence in the control, along the edges of the fragment, of cells with structural nuclei; not only the damaged cells but those adjoining them undergo alteration. In some of these cells the alterations are of reversible nature.

Effect of Ultra-Violet Rays. With considerable dosages of UV (quartz lamp, 40 cm distance, duration 10 minutes or more) cellular alterations take place which are similar to those caused by temperature (heating of the material in this experiment was precluded). Large lumps appear in the nuclei, the nucleoli are surrounded with light areas (Fig. 1, *i*). With lower dosages (3—5-minute exposure) one may attain such a situation that no structural modifications be observed immediately after treatment (Fig. 1, *k*). The effect of UV becomes, however, manifest after some time. When placed into pure water or 1 per cent glucose solution all the cells show after 1 hour structural nuclei (Fig. 1, *l*). The ultraviolet rays produce, consequently, an after-effect; they cause alterations which, being at first invisible, lead the cells to death. These results are in full accord with Alexandrov's observations ([1]).

Effect of Hypertony. Under the influence of hypertonic solutions (glucose, *etc.*) a very fine granulation is seen in the nuclei fixed with OsO_4 (Fig. 1,*m*). One succeds in observing ultramicroscopically under similar conditions the occurrence within the nuclei of luminescent granules (Fig. 2, *e*). The structure thus arising disappears without leaving any trace after the material is transferred to water or 1 per cent glucose solution (Fig. 1, *n*).

Under the influence of the agents discussed above a process of reversible coagulation takes place in the cell nuclei. The structures that appear are undoubtedly formed on the basis of nucleoproteids. In the normal nucleus they are distributed in a diffuse manner and evenly dispersed but fall as soon as the nucleus is damaged, in the form of granules or lumps. A direct idea of this process is furnished by the Feulgen reaction: the normal nucleus is stained diffusely, but in the paranecrotic state the thymonucleic acid becomes condensed into reticular-granular structures. If the process of reversibility is examined in detail, it may be traced on permanent slides that the diffuse coloration of the whole area of the nucleus becomes stronger

according as the number and volume of the lumps decreases. The lumps of the «chromatin» undergo dissolution, dispersion.

It should be noted that although the general reaction of the nuclei to the damaging effect is a monotonous one, *viz.*, structures are formed in them, every agent acts in a specific way. One can easily distinguish, for example, nuclei which were treated with alcohol or heated from such as were placed into hypotonic medium or treated with acid. Under the influence of alcohol or hypotony a fine granular structure arises which has no tendency to fuse to form a net. Under the influence of acids reticular-granular structures are easily formed, while heating leads to the formation of large lumps. The changes caused by alcohol are rather easily reversible, the thermal alterations can be overcome with greatest difficulty, while ultraviolet radiation has an after-effect.

The above data permit of the following conclusions.

1. There is a full correspondence between the vital structure of the nuclei and their structure on permanent slides after fixation with osmic acid.

2. It is possible to demonstrate clearly on slides after fixation with osmic acid the finest alterations of the nuclear structure in damaged cells, and to trace the reversibility of such alterations, in other words, to «fix» paranecrosis.

3. Nuclear structures arise after damage *de novo*. Granules and lumps disappear without trace in the process of reparation, being dispersed during it.

4. While there is a general monotony in the reaction of the cell to the damage, different agents have a specificity of their own.

5. Normal nuclei of the epidermis of *Allium Cepa* and *A. sativum* are endowed with a genuine microscopic homogeneity rather than with a virtual homogeneity caused by the equality of the refractive indices of the nuclear components.

Received
15. VI. 1944.

REFERENCES

[1] Д. Насонов и В. Александров, Реакция живого вещества на внешние воздействия, изд. АН СССР, 1940. [2] D. Nassonov, Z. Zellforsch., **11** (1930). [3] П. В. Макаров, Арх. анат., гист. и эмбр., **19**, 1—2 (1938); **25**, 1 (1940).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

HISTOLOGY

EXPERIMENTS ON HETEROTOPIC TRANSPLANTATION OF THE SKIN

By A. A. BRAUN and G. N. ORLOVA

Communicated by A. A. Zavarzin, Member of the Academy, 27.III.1944)

It has been found by the senior author in his experiments on the structure, development and regeneration of the connective-tissue framework of the skin that (1) the histological structure of the mechanical layer of the skin (which in the higher and lower vertebrates is usually called the reticulum layer of derm and the compact layer, respectively) is regularly developed in all vertebrates; (2) these structures, specific for each species and anatomical region of the skin, are strictly determined and layed down at early stages of development; (3) in the course of regeneration of the wounded region of the skin these specific structures of the mechanical layer of the derm are restored, too. The question arose more than once: what is the behaviour of the skin in transplantation experiments in case the regions to be compared differ from one another in their mechanical tension. It seemed of interest to make clear whether the reconstruction of the mechanical layer of the skin transplanted will take place in the direction specific for its new place, or the structure in question will remain unchanged. To answer this question more than 100 heterotopic auto- and homotransplantations were carried out on axolotls, the age of which varied from 1 to 1.5 years.

Skin fragments were cut out from the head and tail regions with the help of a round punch 8 mm in diameter. The regions were so chosen as to differ as much as possible both in tension and structure of the skin. The reciprocal translocations were made as quickly as possible. As the sizes of wounds were made exactly to correspond to that of the skin bits transplanted, the latter were firmly held by the edges of the wound. Fixation of the pieces transplanted was also facilitated owing to the adhesion by the coagulating tissue fluid. Accordingly, when the axolotls were placed into the water, the transplanted bits did never fall off, but soon grew together with underlying tissues.

Histological examination of the experimental material showed that the structure of the transplants did not change in the new place if the healing was successful. These relations were observed in a score or two of axolotls. A large part of them were nearly one year old; 7 transplants were one year old; and 4 transplants were one and a half year old. Two of the latter are shown in Figs. 1 and 2.

Fig. 1 illustrates the transplantation of the skin piece from the head on to the tail. Fig. 2 shows reciprocal homotransplantation. Both were carried out on axolotls one year old.

Figs. 1 and 2 bear out with utmost clearness that the transplants preserved the same structure that had been peculiar of them before transplantation. As to the mechanical layer of the skin, it should be pointed out that in the transplants it is characterized by a great thickness of the lamellar zone and by strongly developed fibrous structures of the overlamellar zone. The mechanical layer of the tail skin

which is rather poorly developed did not loose its peculiarities throughout its prolonged stay in the head region.

The boundaries of the transplant are the most strictly expressed just in the region of its mechanical layer. In some cases the mechanical layer seems to be cut

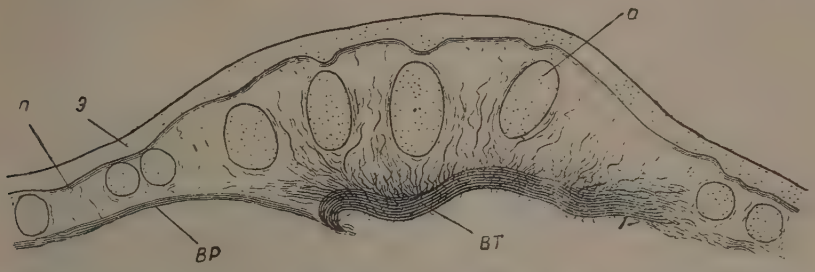


Fig. 1. One-and-a-half year old transplant of skin from head on to tail. Operation on axolotl one year old. *э*, epidermis; *a*, alveolar glands; *n*, subepidermal layer of corium; *ep*, internal lamellar layer of recipient's corium; *eT*, internal lamellar layer of transplant's corium

abruptly at the edge of the bit transplanted (see left end of transplant in Fig. 1). However, the boundaries of the mechanical layer of the transplant are fairly discernible even in the case where this layer is closely connected with that of the recipient by means of the regenerate.

The specific structure of the mechanical layer of the transplant persisted even if a sharp change occurred in the condition of tension. True enough, the position of the transplanted bits is different from that in the normal conditions. The transplant is acted upon by stresses from the scarring regenerate in the formation of which it takes a part along with the host itself. However, only the general configuration

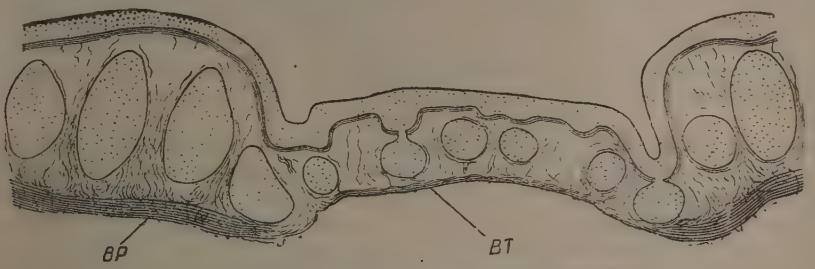


Fig. 2. Same, transplant from tail on to head

of the transplant and the orientation of the fibres over the lamellar zone of the derm in the edge-region of the transplant are determined by the action of the stress in question.

At the same time it may be pointed out that the epithelial structures of the transplants remain unchanged as well. The head and tail alveolar glands differ greatly as to their size. These peculiarities were also observed after transplantation. The same relation was found to be true for the shape and distribution of Leidig's cells. These observation go well to confirm the abundant literary data as to the ability of epithelium and of its derivatives (scales, feathers, hairs) to preserve when transplanted not only their specific structures, but the peculiarities of their initial arrangement, as well.

The A. M. Gorki All-Union Institute
for Experimental Medicine.

Received
27.III.1944.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

PLANT PHYSIOLOGY

VARIATION IN THE CONTENT OF BITTER PRINCIPLE IN THE FRUITS OF HOP, AS INDUCED BY PRE-TREATMENT OF CUTTINGS WITH SALT SOLUTIONS

By I. N. HOLUBINSKY

(Communicated by A. A. Richter, Member of the Academy, 29 V. 1944)

Our preceding note (1) contained a preliminary description of the morphological changes taking place in young plants of hop obtained from such cuttings as had been subjected in spring 1937 to pre-treatment with various salt solutions (largely acid ones) at various concentrations.

As pointed out before (1), we had been stimulated to these experiments by the works of Michurin (3), Baur (4), Keller and Poplavsky (2), and by some other writers who had succeeded in demonstrating that it is possible to obtain modified forms of hop by treating the plant with solutions of certain salts.

The experiments of 1937 which gave definite positive results (appearance of morphological aberrations induced by salt treatment) (1) were repeated in 1939 on a larger scale. The results of the treatment of cuttings proved analogous to those obtained in 1937 *. The solutions applied and the periods of treatment, as well as the methodics of our experiments, have been described in the note cited above[1].

Along with describing the morphological aberrations recorded, in 1939 we proceeded to chemical analyses of the cones, and to a comparison of the respective ratios of bitter substances in untreated plants and in such as had preliminarily been treated with various salt solutions. The chemical analyses of hop fruits taken from plants treated with salt solutions were repeated in 1940.

The results of these experiments did not only fully support our assumptions as to the possibility to induce variation in the chemical composition of the fruit by treating the cuttings with salt solutions, but even surpassed our expectations. Thus, the quantity of the bitter principle in the clone No. 6 did never exceed 12—13 per cent in any of our nurseries. As a result of the salt treatment, separate lots were found to contain up to 19 per cent of bitter substances (2 per cent KBr solution applied).

In 1940 the control plants of the clone No. 6 were found to contain no more than 17.75 per cent of the bitter principle **, while as a result of a treatment with a 3 per cent solution of $KAl(SO_4)_2$ we succeeded in obtaining plants containing up to 25.25 per cent of bitter substances, which is a percentage quite unprecedented in the practice of the agrochemical laboratory of this station. The highest percentage of bitter substances was obtained in our experiments by using KBr and $KAl(SO_4)_2$.

* Similar to the experiments of 1937, in 1939 we made use of clones: in 1937 cuttings of clone No. 2 were treated, in 1939 cuttings of clone No. 6.

** Since the second year after planting the content of bitter substances in the fruits of hop increases, as a rule.

Thus, analyses carried on during 2 years testify to the stability of the character induced by salt treatment, *i. e.* an increased ratio of bitter substances, and to the obtaining of induced new hop varieties, exceeding any variety known heretofore in their resin content. From the results of our experiments it is clear, moreover, that the treatment of hop cuttings with salt solution is a highly promising method which may serve as a means for breeding new valuable varieties of hop.

At the present time we are not in position to answer the question: what is the agent responsible for the sharp increase in the content of bitter substances, observed in the case the cuttings to be planted are subjected to a pre-treatment with various salt solutions. If we take into account that we are dealing here with a number of similar variations, differing in nothing else but the degree of their expression in case different (mostly acid, however) chemical substances are applied, it should be assumed that we are facing here certain variations in the physiological processes taking place in the protoplasm, rather than factor mutations.

In this case we can hardly expect to fix these variations in sexually reproduced individuals. But as hop is a vegetatively reproducible plant, this consideration is scarcely of any importance, while the possibility to increase the percentage of bitter substances by preliminary treatment of cuttings would be an immense achievement and deserves therefore to be paid any possible attention.

Ukrainian Hop-breeding Station.
Zhitomir.

Received
29.V.1944.

REFERENCES

[1] И. Н. Голубинский, Труды Укр. н.-и. станции хмелеводства, в. 3, 80 (1940). [2] Б. А. Келлер и К. М. Поплавский, Тр. Бот. ин-та АН СССР, в. 1, 7 (1933). [3] И. В. Мичурин, Принципы и методы работы, 1939. [4] E. Baur, Z. für indukt. Abst. u. Vererbungslehre, 60, H. 4, 467 (1932).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

PLANT PHYSIOLOGY

LATICIFEROUS SYSTEM AS ONE REGULATING THE COURSE AND DIRECTION OF BIOCHEMICAL PROCESSES IN PLANTS

By A. A. NICHIPOROVICH

(Communicated by A. N. Bach, Member of the Academy, 18.VI. 1944)

In spite of the fact that studies on the physiological rôle of the laticiferous vessels have a nearly half-centenary history, there is still no unanimity of opinion in this problem.

Some students consider rubber as a typical excrete and the laticiferous vessels, as receptacles of the excrete, denying any vital functions appropriate to them ([1–3]).

Others consider the laticiferous vessels as ducts serving for the transport of materials, or as magazines for the storage of water and food, or as organs regulating the water economy of the plants ([4–7]). Some students belonging to the latter group regard rubber as a reserve stuff which is spent by the plants during the metabolic process ([8–10]).

Naturally enough, a correct solution of this problem is not only of theoretical importance, but has an important practical bearing as a starting point for working out means of enhancing rubber content and productivity of the rubber plants.

In our two preceding papers ([11], [12]) we have shown, first, that the laticiferous system of kok-saghyz and krym-saghyz is a unitary system which connects by continuous channels the assimilating and evaporating organs with the storing and water-absorbing organs; secondly, as an extension of the works by Arisz ([13]), Frey-Wyssling ([14]), Spenser ([15]) and others, we have shown that laticiferous vessels are important agents regulating the water regime of the plant since, on the one hand, neighbouring cells can absorb water from them and, on the other hand, by lowering the turgor they can themselves absorb water from the adjoining cells of the tissues containing more water, and thus influence the transport of water, its uptake and the course of its evaporation. In this manner with the aid of the laticiferous vessels the equilibrium of the water regime of the different organs and tissues can be rather easily established or stabilized.

Along with this the state of the water balance of the cells, tissues and organs of the plants is an important agent which determines the intensity and direction of the biochemical processes, acting upon the energy of the respiration processes ([16]), upon photosynthesis ([17–19]), upon the character of the carbohydrate and protein exchange ([20–24]), as well as upon the direction of the fermentative processes ([25]), *etc.* The fact alone that with the aid of the laticiferous vessels other influences which determine the water regime of the cells are transmitted from one tissue to another, makes us suppose that the laticiferous vessels are at the same time the transmitters of stimuli for alterations in the direction and kind of a number of biochemical processes.

The following facts are in favour of this assumption: Bobilioff ([26]) has proved that in the cortex of *Hevea* abundant deposition of starch is localized mainly in those cells which immediately adjoin the laticiferous vessels. During the periods of tapping, when the turgor in the vessels is reduced and they absorb intensely water from the neighbouring cells and regenerate latex and rubber, the starch content within the parenchymatous cells is reduced considerably.

The same phenomenon has been proved with regard to inuline in the roots of tau-saghyz which were subjected to tapping according to the method of Mazanko ([27]).

A similar picture was observed by us in the roots of kok-saghyz subjected to a prolonged withering in pots. In this case there was a considerable decrease in inuline in the laticiferous vessels and in the cells immediately adjoining them.

Chernykh ([28]) observed the disappearance of inuline from the cortical parenchymatous cells adjoining laticiferous vessels in the roots of wild growing plants at the time of intense formation of rubber.

It should be noted at the same time that in the natural growth of this rubber-plant the period of intense rubber accumulation coincides with the increase in drought and heat and precedes the entering of the plant into the summer-rest stage.

All the above facts should be taken to mean that laticiferous vessels are active organs which strongly influence the biochemical reactions in the cells and tissues adjoining them, and that one of the means of interaction between the laticiferous vessels and the neighbouring cells is probably the disturbance of equilibrium in their water regimes, and the modification of the direction of the action of the sorbing power and mutual water exchange.

In order to study the effect of the filling of the laticiferous vessels with latex upon other physiological processes, in particular upon respiration, we have conducted on the roots of kok-saghyz some experiments which proved that this process, too, is strongly influenced by the state of the laticiferous vessels. For the tests parallel samples of roots were taken which were treated in two ways: fresh turgid roots of some plants were cut into parts 7 cm long with the aim to avoid flowing out of the latex. The roots were cut across with a sharp hot knife. This caused coagulation of the latex at the moment when cutting was done, and closed the vessels. The roots of the other group were cut into pieces 9 cm long with the aid of a cold knife. Then discs 1—2 mm thick were sliced from the two ends of the root piece, until latex ceased to appear on the surface of the wound. In this case, as shown by analysis, about 25 per cent of the total rubber content was extracted from the roots. Finally, the two ends of the root piece were cut with a hot knife in order to make conditions similar in the two groups of roots.

Two samples from the first and second groups each were placed into chambers to record respiration; its intensity was studied in an air stream passing through absorbers (system of A. A. Richter, Member of the Academy ([29])).

The following are the result of the determinations made by N. G. Vassilieva (Table 1).

Table 1

Roots	Energy of respiration in mg CO_2/hour per 100 g of roots							
	During first hour		During second hour		Next day		Third day	
	samples	average	samples	average	samples	average	samples	average
Control samples	23.43 25.93	24.68	25.93 28.48	27.20	23.74 23.06	23.40	19.55 18.28	18.92
Roots with partly extracted latex	21.25 21.05	21.15	21.25 21.09	21.17	25.93 24.77	25.35	22.53 22.31	22.42

As may be seen from Table 1, partial evacuation of the laticiferous vessels materially alters the respiratory intensity of the roots and the dynamics of this process in the course of time; while in the first hours its intensity is less in the roots with partly evacuated laticiferous vessels, later the relation between the control and the evacuated roots is the opposite.

In other instances samples of the roots weighing 30 g each were placed for the determination of gaseous exchange into respirators designed by Jones ([30]). The apparatus consisted of 12 containers immersed into a water thermostat. The gaseous exchange was recorded volumetrically. The samples were distributed among the containers in the following manner: three containers were filled with samples of the first group. Solution of KOH was put into the containers in order to absorb the carbon dioxide produced by the roots. In the three other containers roots of the same group were placed, but no alkali was added: each of the remaining three containers were loaded in the same way with roots of the second group.

The experimental scheme described above made it possible to record not only the respiration intensity, but the respiration coefficients of the roots tested, too.

It should be noted that the general indices of the intensity of respiration in closed containers proved to be lower than in the case of those obtained in the stream of air (Table 1); the differences in respiration intensity between the control and experimental samples were, moreover, not so sharp as in the preceding experiment. However, in all the experiments without any exception, the respiration coefficients of the experimental partly evacuated roots proved to be considerably less than those of the controls, and this difference should be considered fully significant.

The results of these experiments which were carried on under the cooperation of L. A. Ostapenko are summarized in Table 2.

Table 2

Date and time of the experiment*	Control samples			Samples of roots with partly removed latex		
	Produced CO_2 cm^3/hour per 100 g	Absorbed O_2 cm^3/hour per 100 g	CO_2/O_2	Produced CO_2 cm^3/hour per 100 g	Absorbed O_2 cm^3/hour per 100 g	CO_2/O_2
26.IX 13h10′—17h10′	5.37	5.67	0.95	4.37	5.68	0.77
27.IX 9 50 —11 30	4.89	5.28	0.93	4.04	5.60	0.72
27.IX 16 00 —17 00	5.62	5.94	0.95	3.56	6.02	0.59
28.IX 9 50 —13 50	4.83	5.36	0.90	4.06	5.08	0.80
30.IX 13 45 —16 45	3.93	4.34	0.91	3.30	5.18	0.64
1.X 9 40 —13 40	3.43	3.89	0.88	3.51	4.59	0.76
17.X 15 30 —17 30	5.24	5.44	0.96	4.37	5.91	0.74
18.X 10 30 —12 30	5.61	5.94	0.94	6.77	8.32	0.81

* The faucets of the respiratory containers were opened every hour and recording was started anew. In the table are presented averages from several successive readings in three parallel containers of each series.

Without making any suggestions as to the causes of changes of the respiratory coefficient of the roots which were partly deprived of latex, we shall limit ourselves by the statement that there exists an effect of the state of the laticiferous vessels upon a number of important biochemical processes and, in particular, upon the processes of hydrolysis of highly molecular carbohydrates, upon the respiration intensity and respiratory coefficients. As the laticiferous vessels which are in intimate relations with the living parenchymatous cells are subject to strong influences of the latter as regards water content, turgor, direction of the process of

water exchange, and they connect different organs and tissues by continuous channels accomodating for an easy transport of fluids, they (apart from other factors) may themselves act as important agent ensuring the coordination of the vital processes in the various parts of the plant, facilitating its life activity as an unity.

Received
18.VI.1944.

REFERENCES

[1] W. Bobilioff, Anatomie en physiologie van Hevea brasiliensis, Batavia, 1930. [2] A. Frey-Wyssling, Die Stoffausscheidung der höheren Pflanzen, Berlin, 1935. [3] T. Petch, The Physiology and Diseases of Hevea brasiliensis, London, 1911. [4] G. Haberlandt, Physiologische Pflanzenanatomie, Leipzig, 1918. [5] H. Molisch, Studien über den Milchsaft und Schleimsaft der Pflanzen, Jena, 1901. [6] H. Fickendey, Tropenpflanzen, **14**, 481 (1910). [7] Е. Гаузер, Латекс, М.—Л., 1932. [8] C. Harries, Untersuchungen über die natürlichen und künstlichen Kautschukarten, Berlin, 1914. [9] D. Spence, Trans. Inst. Rubb. Ind., **11**, 7, 119 (1935). [10] Ф. Мазанко, Яровизация, **3** (36), 12 (1941). [11] A. Nichiporovich, C. R. Acad. Sci. URSS, XL, No. 6 (1943). [12] A. Nichiporovich, ibid., XLII, No. 4 (1944). [13] W. Arisz, Arch. Rubbercult. Nederl. Indie, **12**, 220 (1928). [14] A. Frey-Wyssling, ibid., **16**, 9, 285 (1932). [15] H. Spenser, Ann. of Bot., **9**, 237 (1939). [16] W. Iljin, Flora (N.-F.), **116**, 379 (1923). [17] W. Iljin, Flora, **116**, 360 (1923). [18] А. Алексеев, Бот. журн. СССР, **20**, 227 (1935). [19] В. Бриллиант, Изв. Гл. бот. сада, **24**, 1 (1920). [20] А. Курсанов, В. Благовещенский (мл.), М. Казакова, Бюлл. Моск. о-ва испыт. природы, **42** (2), 3, 171 (1933). [21] С. Д. Львов и С. С. Фихтенгольц, Эксперимент. ботаника, **2**, 149 (1936). [22] И. Васильев, Тр. пр. бот., ген. и сел., **27**, 3 (1931). [23] W. Iljin, Planta, **10**, 170 (1930). [24] K. Mothes, Ber. deutsch. bot. Ges., **46**, 59 (1928). [25] Н. Сисакян, Биохимическая характеристика засухоустойчивости растений, изд. АН СССР, М.—Л., 1940. [26] W. Bobilioff, Arch. Rubbercult. Nederl. Indie, **4**, 3, 71 (1920). [27] Ф. П. Мазанко, Сборн. работ Биохимия и физиология каучуконосных растений, М., 1939. [28] С. Липшиц, Новый каучуконосный одуванчик Taraxacum kok-saghyz, М.—Л., 1934. [29] A. Richter, C. R. Acad. Sci. URSS, XI, 7 (1936). [30] W. Jones, Ann. of Bot., **43**, 172, 841 (1929).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

PLANT PHYSIOLOGY

FLOWERING IN DIFFERENT PLANT SPECIES AS A RESPONSE TO NITROGENOUS FOOD

By M. CH. ČAJLACHJAN

(Communicated by A. A. Richter, Member of the Academy, 15.XI. 1944)

In the course of his studies (²) of the effect produced by nitrogenous fertilizers upon the development of plants, the present author became aware of the fact of different plant species responding in a different way to a reinforced mineral nitrogenous diet. This observation being both of theoretical and practical importance, it made the author extend his studies to other geographical conditions, and to carry out a number of experiments in the extreme south of this country.

The experiments were carried on during the vegetation period of 1942—1943 in Erevan, at a green-house of the Armenian Agricultural Institute.

Experiments involving wheat (*Triticum vulgare* var. Delphi), maize, sunflower, Egyptian cotton (*Gossypium barbadense*) and different varieties of the American cotton-plant (*Gossypium hirsutum*) were carried out according to the method of sand-cultures in glass vessels with a capacity of 4 kg of sand; the plants were given full Prianishnikov solution, except ammonium nitrate which was supplied to the vessels after the following sheme: 1) without nitrogen, zero (0 N), 2) half a nitrogen ration (0.5 N), 3) one ration of nitrogen (1 N) and 4) two nitrogen rations (2 N). When filling the vessels with sand, the plants of the three last runs were given half a ration of nitrogen each, the third and fourth runs were kept in the subsequent at a reinforced nitrogenous diet (addition of 0.5 N). In the experiments with the American cotton varieties, nitrogen was supplied to the plants of all runs at once, before planting. Each experimental run was represented by two vessels of each crop under test. Humidity was maintained at 70 per cent to total moisture capacity of the sand throughout the period of experiments.

Data on the development of the plants are to be found in Table 1.

The wheat-variety «Delphi» was the first to start earing and blooming when entirely withheld from nitrogen; when given 0.5 N, 1 N or 2 N, these stages would set in later and about at the same time; the ripening of the ears proceeded the later, the higher was the ration received by the plant (plants kept on 0 N failed to develop any seed in their ears). An opposite reaction was observed in maize, sunflower and the Egyptian cotton-plant: these plants responded to increased nitrogen diets by passing to reproductive development at an earlier date. Maize failed to develop inflorescences when kept on nitrogenless diet; in runs kept on 0.5 N, 1 N and 2 N the number of inflorescences was 1, 3 and 4—5, respectively. The sunflower showed early ripening when given 2 N and 1 N; Egyptian cotton, after budding at 1 N and 2 N, did not pass to the subsequent development stages.

From the data on budding of the cotton-plant (Table 1) it may be seen that the ability of the cotton-plant to pass to the stage of reproductive development is regularly correlated with the earliness of the variety: the later a given variety

Table 1

Growth and Development of Plants as Affected by Nitrogenous Diet (Sand Cultures)

Plants	Flowering				Height of plants (in cm)				Dry weight of one plant (in g)			
	0 N	0.5 N	1 N	2 N	0 N	0.5 N	1 N	2 N	0 N	0.5 N	1 N	2 N
Wheat	6.VII	9.VII	9.VII	9.VII	56	64	81	64	0.61	1.58	2.90	1.78
Maize	none	26.VII	19.VII	20.VII	51	65	91	93	3.00	8.30	22.20	24.30
Sunflower . .	1.VIII	23.VII	22.VII	20.VII	79	117	121	129	1.54	6.35	6.55	7.82
Egyptian cotton	none	none	26.VII	24.VII*	21	30	31	36	2.16	4.61	6.70	9.11
American cotton «Standard 915», early variety «084», medium earliness . .	18.VIII*	18.VIII*	18.VII*	22.VII*	20	37	44	46	5.97	11.82	26.21	32.05
«Ferghana 031», late variety	none	20.VIII*	19.VII*	20.VII*	28	51	57	55	7.86	38.37	39.86	56.31
	none	none	24.VII*	3.VIII*	30	57	65	63	7.42	27.87	42.05	62.00

* The plants developed flowering buds only.

comes into ripening, the higher the lower limit of nitrogen content in the substratum, which is necessary for the plant in order to break into budding.

Experiments on alfalfa (*Medicago sativa*), clover (*Trifolium repens*), red garden-bean (*Phaseolus vulgaris*), the tobacco variety «Samsun» (*Nicotiana tabacum*) and early cotton variety «Standard 915» (*Gossypium hirsutum*) were carried out in earthern pots about 5 inches high, containing rather poor soil mixed with sand at a ratio 2 : 1. The experiments were carried out in 4 runs: 1) long natural day and unfertilized soil (—N); 2) same, and nitrogenous diet (+N); 3) short 10-hour day and unfertilized soil (— N); 4) same, and nitrogenous diet (+N). The curtailing of the day was attained by means of a veneer-chamber to which the plants were transferred after 6 p. m. till 8 a. m. of the next day. Nitrogen was supplied before sowing, on May 17th 1943, and at five subsequent dates: on the 4th of June, 19th of June, 5th of July, 17th of July and 10th of August. The dose supplied was half a ration of nitrogen of Prianishnikov's nutrient solution (0.12 g NH_4NO_3 per 1 kg of soil). Watering up to total moisture capacity of soil; replication, twofold.

Data illustrating the flowering of the plants, their height, dry weight, *etc.*, are to be found in Table 2.

Table 2

Growth and Development of Plants as Affected by Nitrogenous Diet (Soil Cultures)

Plants	Flowering				Dry weight of one plant (in g)				Weight of nodules per one plant (in g)			
	Long day		Short day		Long day		Short day		Long day		Short day	
	—N	+N	—N	+N	—N	+N	—N	+N	—N	+N	—N	+N
Alfalfa	12.VII	22 VII	none	none	1.01	1.07	0.41	0.62	0.026	0.023	0.015	0.004
Clover	10.VIII	none	none	none	2.90	3.40	0.80	2.50	0.323	0.234	0.072	0.070
Garden-bean .	3.VII	4.VII	4.VII	3.VII	2.60	3.10	1.10	1.50	0.253	0.204	0.020	0.007
Cotton (early)	16.VIII	16.VIII	12.VIII	9.VIII	5.20	8.30	—	4.40	—	—	—	—
Tobacco . . .	22.VIII	26.VIII	13.VIII	18.VIII	26.10	29.20	22.10	16.20	—	—	—	—

Alfalfa and clover which are typical long-day plants, did not flower unless given a long day; the effect of nitrogenous additional rations was to delay the flowering of these plants. As to clover, this plant failed to blossom throughout the experiment if grown on fertilized soil. A somewhat different response was observed on the part of garden-peas: these plants started flowering at about the same date both with long and short day, in experiment and control pots. Early cotton-plants set in flowering at an earlier date when kept under short-day regime; under these conditions additional nitrogen diet went to accelerate flowering, whereas it failed to produce any appreciable effect under long-day conditions. In the case of tobacco additional nitrogen diet accelerated flowering considerably when given under long-day conditions, while under short-day conditions its effect was to delay flowering, which even accounts for a difference in the photoperiodic reaction shown by the plant when grown in unfertilized or fertilized soil.

In the experiments on leguminous plants here described the effect of mineral nitrogen was combined with that of the nitrogen fixed by nodule bacteria. From the figures characterizing the dry weight of the plants it may be inferred, however, that the more intense development of nodules upon the roots of control clover and bean-plants does not compensate for the influence of additional nitrogen supply.

Summing up the results of the experiments here described we become aware of the fact that according to how they respond by blooming to the alteration in their nitrogen diet, the species investigated may be classed into groups analogous to those observed by us for other species ([2]), *viz.*: 1) wheat, alfalfa and clover are made to break into blossom earlier when kept on meagre nitrogenous diet, *i. e.* behave in the same way as did oats and mustard in our preceding experiments; 2) maize, sunflower, tobacco and various species of cotton start blooming earlier under a normal or excessive nitrogen supply, *i. e.* behave in the same way as perilla, millet, lupine, lettuce and double-flowered chrysanthemum tested before; 3) garden-beans start flowering at the same date, irrespective of the nitrogen supply, like buckwheat, soya and hemp that were tested in our earlier experiments. Within the second group, maize and different varieties of the American cotton-plant stand somewhat apart, having a lower limit of nitrogen requirement for their reproductive development than that of sunflower and tobacco.

Thus, along with a reaction that is common to all plant species which respond to alterations in their nitrogen diet by corresponding changes in growth, increase by volume and weight, as well as by the quantity of their ultimate yield in fruit and seed, there exists another reaction, the one observed in the reproductive development of the plants, and this reaction differs as to the species. If data available in literature are viewed at this angle, they are found to support the existence of such specific differences in the reaction of plants belonging to different species.

Auchtor and Harley ([4]) found that soja of the variety «Biloxy» when grown on a mixture of $^2/_3$ clay and $^1/_3$ manure, with a subsequent watering of the soil with nitrogen-containing solutions, set in flowering at the same time as the plants of the same variety grown on a sand poor in nitrogenous substances, whereas pepper, lettuce and radish were delayed in flowering when grown on a substratum low in nitrogen. In Nightingale's experiments ([8]) *Salvia splendens* which does not flower, as a rule, unless exposed to short photoperiods, broke into blossom under long-day conditions when transferred to a diet poor in nitrogen; Borodina ([1]), reporting on the water cultures of Mongolian barley var. *trifurcatum* and of the Saratov millet, says that by excluding nitrogen from the nutrient solution, barley was made to start earing at an earlier date, while the appearance of panicles in millet was found to be somewhat delayed by the same procedure, which has in the subsequent been confirmed by Razumov. Schapelle ([10]) found that flowering in spinach (var. Bloomsdale Savoy-leaved) and raddish could be postponed by adding nitrate solution to the soil contained in the pots in which these plants were grown. Later on results obtained with another variety of spinach («Longstanding Bloomsdale») carried out by Knott ([6]) went to support these observations. Roberts and Esther Struckmeyer ([11]) who were studying the influence of temperature upon the photoperiodic response of various plants, describe an experiment in which *Tagetes* begun flowering in

soil and refused doing so in sand, while *Clarkia*, lettuce and geranium started flowering at an earlier date when grown in sand. Edith Neidle (7) demonstrated that *Xanthium pennsylvanicum* breaks into blossom the earlier, the higher the substratum is in nitrogen. The most comprehensive work on the subject is that by Denffer (5) who, experimenting upon water-cultures, has demonstrated that hard wheat (var. «Heines»), four-rowed barley (var. «Heines») and *Iberis amara* are stimulated to earlier blossoming by withhelding them from nitrogenous food, whereas mohar ((*Setaria italica*), *Chrysanthemum Mepho*, *Tinantia fugax* and *Kalanchoe blossfeldiana* start flowering at a later date.

The experimental data cited in this work and in the preceding papers by the same author (2, 3), supplemented by data by other authors cited above, allow the plant species investigated to be classed among the following groups, according to the way in which they respond by flowering to changes in their nitrogenous diet.

Group I. Nitronegative plants, entering into blossom sooner when either entirely withheld from nitrogen, or supplied with reduced rations of nitrogen, or grown on a soil poor in nitrogenous substances. Among these should be classed: 1) wheat (different varieties of *Triticum vulgare*); 2) barley (*Hordeum vulgare*), different varieties; 3) oats (*Avena sativa*); 4) mustard (*Sinapis alba*); 5) spinach (*Spinacia oleracea*), different varieties; 6) alfalfa (*Medicago sativa*); 7) clover (*Trifolium repens*); 8) salvia (*Salvia splendens*); 9) *Clarckia elegans*; 10) geranium (*Pelargonuim hortorum*); 11) *Iberis amara*.

Group II. Nitropositive plants, entering into blossom at an earlier date with a normal or increased nitrogen supply, or when grown on a soil high in nitrogenous substances. Among these plants should be classed: 1) millet (*Panicum miliaceum*), different varieties; 2) Indian corn (*Zea Mays*); 3) mohar (*Setaria italica*); 4) *Perilla nankinensis*; 5) sunflower (*Helianthus annuus*); 6) tobacco (*Nicotiana tabacum*); 7) cotton (different species and varieties of *Gossypium hirsutum* and *G. barbadense*); 8) pepper (*Capsicum annuum*); 9) lettuce (*Lactuca sativa*); 10) lupine (*Lupinus angustifolius*); 11) different species of *Chrysanthemum indicum* and *Chr. Mepho*; 12) *Tagetes erecta*; 13) *Xanthium pennsylvanicum*; 14) *Tinantia fugax*; 15) *Kalanchoe blossfeldiana*.

Group III. Nitroneutral plants, whose flowering date remains constant, irrespective of the nitrogen ration supplied, on soils low in nitrogenous substances, or high in them. Among these should be classed; 1) buckwheat (*Fagopyrum esculentum*); 2) hemp (*Cannabis sativa*); 3) soja bean (*Soja hyspida*), different varieties; 4) garden-bean (*Phaseolus vulgaris*).

The K. A. Timiriazev Institute for Plant Physiology.
Academy of Sciences of the USSR.
Armenian Agricultural Institute.

Received
15. XI. 1944.

REFERENCES

[1] И. Н. Бородина, Тр. по прикл. бот., ген. и сел., 5 (1931). [2] М. Х. Чайлахян, Изв. Арм. фил. АН СССР, № 9–10 (1942); C. R. Acad. Sci. URSS, XLIII, No. 9 (1944). [3] M. Ch. Čajlachjan, C. R. Acad. Sci. URSS, XLIV, No. 8 (1944). [4] E. C. Auchtor and C. P. Harley, Proc. Am. Soc. for Horticultural Science, 21 (1924). [5] D. Denffer, Planta, 31, H. 3 (1940). [6] J. E. Knott, Plant Physiology, 15, No. 1 (1940). [7] Edith Neidle, Bot. Gaz., 100, No. 3 (1939). [8] G. F. Nightingale, Wiscons. Agr. Exp. Sta. Res. Bullet., No. 74 (1927). [9] R. H. Roberts and B. E. Struckmeyer, J. of Agricult. Res., 56 (1938). [10] N. A. Schapelle, Cornell. Univ. Agric. Exp. Sta., Memoir No. 185 (1936).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 2

PHYSIOLOGY

EVALUATION OF THE AMOUNT OF BLOOD DISPLACED IN THE EXPERIMENT OF VALSALVA

By A. M. DAMIR

(Communicated by A. I. Abrikossov, Member of the Academy, 26.IV.1944)

1. The Valsalva ([2]) experiment was carried out by us with clinically sound persons, *i. e.* such who had no complaints at the moment nor registered any diseases in the past. The person under experiment was fixed on platform P of the apparatus which we had devised ([1]) for determining the gravity centre Z of the human body. The head end P_h of platform P was burdened by the body somewhat more than the feet end P_f. The equilibrium of platform P was brought about by the supporting power w applied to the terminal point K of the head end P_h of the platform, lying at a distance of 104.4 cm from the fulcrum F. The force w was registered by the index of the circular Berkel balance R. In the III and IV stages of the Valsalva experiment ([2]) all the body organs are immobile. The intrathoracal and intraabdominal pressure arising at the beginning of stage III of the experiment which reaches its maximum at the end of this stage, produces the displacement of great masses of blood. This leads to displacements of the body's gravity centre, first towards the head, later in the direction of the feet. The task we had set ourselves was to evaluate quantitatively the blood masses displaced according to the displacements of the gravity centre Z.

The pressure arising at stage III flattens the *vena cava superior* as well as those parts of all the veins of large and medium gauge in the thorax which lie in the vicinity of the heart. This isolates the heart from the access of blood to it from the body parts lying upwards of the diaphragm. This means that its subsequent work results in driving the blood into these parts without its returning back. But from the body parts lying below the diaphragm blood continues to flow into the heart during stage III, for although the pressure tends to compress the *vena cava inferior* as well, the same pressure squeezes the liver against the dome of the diaphragm and forces the blood out of it into the *vena cava inferior*, creating in it provisionally so powerful an internal pressure which overpowers the external one and does not allow the vein to be flattened. This continues until all the blood of the liver is spent; but at the moment when its quantity becomes already insufficient, the *v. cava inferior* is flattened out, and the access of blood from the body parts lying on this side of the diaphragm is also stopped. Thus, the heart is completely isolated and the next stage IV of the Valsalva experiment begins. The persisting work of the heart results in driving all the blood yet available to it, *i. e.* that which remains in the small circulation, through the arterial system to the periphery. By the end of stage IV the entire small circulation including the lungs is emptied, and the heart is completely drained. The result of the draining of the thorax and abdomen is the repletion of the peripheral veins with blood. Since the capacity of the veins of the lower limbs and of the lesser pelvis is greater than that of the

veins of the upper limbs and the head, the supporting power w must in the course of stage III decrease monotonously from value w_3 which it had at the end of stage III to w_4 which it reaches at the end of stage IV. In connexion with this the gravity centre Z of the body is displaced in the direction of the feet from position Z_3 to position Z_4. It should be noted that in the course of stage III the supporting power w experiences a monotonous rise from value w which it had at the end of stage II to value w_3 which it reaches at the end of stage III. In connexion with this the gravity centre Z of the body is displaced towards the head from position Z_2 to position Z_3. The explanation of this is that though during stage III we have to deal with two opposed factors: the driving of blood by the semi-isolated (from above) heart to the periphery and its supply with blood from below *via* the *vena cava inferior*, — it is the latter factor that evidently obtains the upper hand.

As the chief interest of the Valsalva experiment consists in the almost complete draining of both the thoracal and abdominal cavities achieved by it, it is natural to try to find the amount of blood eliminated from them and driven off to the periphery. We term the amount of blood displaced in the course of stages III and IV of the Valsalva experiment from the thoracal and abdominal cavities to the periphery the quantitative value X of the Valsalva effect.

2. Approximate Estimation of the Mass of Displaced Blood. Let us consider the last moment of stage IV of the Valsalva experiment: the thoracal and abdominal cavities are drained, and the supporting power w has reached value w_4. But during stage III the blood from the abdominal cavity was transferred through the *vena cava inferior* into the thoracal cavity. Considering the extreme brevity of stage III (2 seconds only), we may ignore the leakage of blood to the arterial system during this interval of time. Therefore in evaluating the mass of blood displaced to the periphery we may take into account the monotonous decrease in weight w from w_3 to w_4.

Let us denote by X the mass of blood which fills the abdominal and thoracal cavities at a given moment of time t. With good approximation we may consider the gravity centre x of mass X as coinciding with the gravity centre $Z^{(*)}$ of the upper half of the body. By the term «upper half of the body» we mean that part of the body of the person subjected to the experiment which lies between the fulcrum F and point K. The mean distance Fx between points F and x (or $Z^{(*)}$) is 35 cm.

Let us introduce *explicite* the following hypothesis: in the Valsalva phenomenon the blood X is distributed throughout the human body in such a way that the gravity centre x^* of the distributed mass X^* of blood X coincides with the fulcrum F of the apparatus.

This hypothesis is in many cases sufficient, since practically we may assume that blood X which leaves the trunk is equally distributed between the upper and lower halves of the body. In this case the phenomenon of Valsalva consists simply in a subtraction of mass X from the body, and the disturbed equilibrium of platform P is restored by the vertical power $w_3 - w_4$ applied at point K and directed downwards. From the equation of moments $XFx = 104.4\ (w_3 - w_4)$ we find

$$X = 104.4\,(w_3 - w_4)\,/Fx \qquad (1)$$

In the «mean» case: $w_3 - w_4 = 450$ g, and $Fx = 35$ cm. It follows that $X = 1342$ g. Hence, keeping in mind the mean amount of blood in the human body, we conclude that in the course of the Valsalva experiment and, consequently, in the course of any intense muscular strain, approximately $^1/_3$ of the total blood mass is transferred from the trunk to the periphery.

Moreover, the mean value of X determined by us shows that an active part belongs in the Valsalva effect to the deposited blood.

3. Exact Evaluation of the Mass of Displaced Blood. This evaluation is obtained if we substitute the *a priori* natural and regularly corroborated in the experiments principle of proportionality for the hypothesis assumed in the preceding paragraph.

The Principle of Proportionality. In the phenomenon of Valsalva the blood is distributed throughout the human body proportionally to the masses of its parts.

Let us denote by X_h, X_a, X_l the amounts of blood X which have been displaced to the head, arms and legs, respectively. Let M_h, M_a, M_l signify the respective masses of these parts of the body, and x_h, x_a and x_l—their respective gravity centres during the usual position of the person subjected to the experiment on the platform. According to the above principle we shall have:

$$X_h = kM_h, \quad X_a = kM_a, \quad X_l = kM_l$$

where k is the unknown coefficient of proportionality. By adding together X_h, X_a and X_l we obtain:

$$X = X_h + X_a + X_l = k(M_h + M_a + M_l)$$

Turning to the equation of moments for the end of stage IV

$$104.4(w_3 - w_4) + kM_h Fx_h + kM_a Fx_a = kM_l Fx_l$$

we find from it the unknown coefficient of proportionality k and introducing it into the formula written above for X obtain:

$$X = \frac{104.4(w_3 - w_4)(M_h + M_a + M_l)}{M_l Fx_l - M_h Fx_h - M_a Fx_a} \tag{2}$$

This formula is considerably simplified by introducing the trunk mass M and its gravity centre x_t, denoting by M the mass of the entire body. It is evident that $M_h + M_a + M_l = M - M_t$. On the other hand, the equation of the moments previous to the Valsalva experiment is

$$M_h Fx_h + M_a Fx_a + M_t Fx_t = M_l Fx_l$$

Hence formula (2) may be substituted by

$$X = 104.4(w_3 - w_4)(M - M_t)/M_t Fx_t \tag{3}$$

analogous in appearance to formula (1). After transcribing formula (3) in the shape of

$$X = [104.4(w_3 - w_4)/Fx_t](M - M_t)/M_t \tag{3'}$$

we may now turn to the investigations of Fischer [3] and Bernstein [4].

According to these authors, we have the following values: for the head — $M_h/M = 0.07060$ and for the limbs: M (of hips)$/M = 0.24970$; M (of shins)$/M = 0.09462$; M (of feet)$/M = 0.02786$; M (of arms)$/M = 0.05264$; M (of forearms)$/M = 0.03638$; M (of hands)$/M = 0.01284$. By adding these together we obtain $(M - M_t)/M = 0.54464$, and accordingly $M_t/M = 0.45536$. Hence the last factor in formula (3'), $(M - M_t)/M_t$, is simply a definite value $54464/45536 = 1.19606$ .... and the formula (3') may now be transcribed as

$$X = 1.19606 \cdot 104.4(w_3 - w_4)/Fx_t \tag{4}$$

We see that the exact formula (4) gives a result which exceeds that of the approximate formula (1) by $^1/_5$.

First Moscow State Medical Institute.

Received 20. I. 1944.

REFERENCES

[1] A. M. Damir, C. R. Acad. Sci. USSR, XLIV, No. 2 (1944). [2] Idem, ibid., XLV, No. 2 (1944). [3] R. A. Fischer, Statistical methods for research workers, London, 1930. [4] Н. Бернштейн, Физиологич. труды, 1934.